Chapter 1

DARK MATTER: EARLY CONSIDERATIONS

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Abstract   A review of the study of dark matter is given, starting with earliest studies and finishing with the establishment of the standard Cold Dark Matter paradigm in mid 1980-s. Particular attention is given to the collision of the classical and new paradigms concerning the matter content of the Universe. Also the amount of baryonic matter, dark matter and dark energy is discussed using modern estimates.

Keywords: Dark matter; galaxies; clusters of galaxies

1.   Introduction

Dark matter in the Universe can be described as the matter which has practically zero luminosity and its presence can be detected only by its gravity. Historically, the first modern study of the possible presence of dark matter goes back to 1915, when Öpik (1915) determined the dynamical density of matter in our Galaxy in the Solar vicinity. The same problem was investigated by Oort (1932, 1960), Kuzmin (1952a, 1955) and more recently by Bahcall (1985) and Gilmore, Wyse & Kuijken (1989). Modern data suggest that there is little evidence for the presence of a large amount of local dark matter in the Solar vicinity. If some invisible matter is there, then it should be in the form of brown dwarfs, jupiters or similar compact baryonic objects.

A different type of dark matter is found around galaxies and in clusters of galaxies. The first evidence for the presence of such global dark matter was given by Zwicky (1933) from the dynamics of galaxies in the Coma cluster. The presence of dark matter in clusters was questioned, and an alternative solution to explain large velocities of galaxies in clusters was suggested by Ambartsumian (1958) – the instability of clusters of galaxies. However, the evidence for the presence of invisible matter in systems of galaxies accumulated, first for our Local Group of galaxies (Kahn &Woltjer 1959), and thereafter for all giant galaxies (Einasto, Kaasik & Saar 1974, Ostriker, Peebles & Yahil 1974). These results were questioned by Burbidge (1975), Materne & Tammann (1976). Independent determination of rotation velocities of galaxies at large galactocentric distances (Rubin, Ford & Thonnard 1978, 1980) confirmed previous results on the presence of dark halos or coronas around galaxies. The nature of dark matter around galaxies is not clear. Initially it was assumed that it consists of hot gas (Kahn & Woltjer 1959, Einasto 1974b). Mod-

ern data favour the hypothesis that dark matter around galaxies is non-baryonic, either neutrinos or some weakly interacting massive particles, such as axions. The neutrino-dominated dark matter is called hot, since neutrinos move with very high velocities. The other type of dark matter is called cold, as particle velocities are moderate. The cosmological model with cold dark matter (CDM) was suggested by Blumenthal et al. (1984). This model is presently accepted as the standard. With the establishment of the cold dark matter concept the early period of the study of dark matter was completed.

Excellent reviews of the dark matter problem have been given by Faber & Gallagher (1979), Trimble (1987), Turner (1991) and Silk (1992), alternatives to dark matter have been discussed by Sanders (1990). In this report I describe how astronomers developed step-by-step the concept of dark matter. Such process is typical for the formation of a new paradigm in our understanding of the Universe. Particular attention is given to the work on galactic modelling which has lead us to the understanding of the structure of stellar populations and the need for a new invisible population of dark matter in galaxies. The Power-Point version of the present report is available on the web-site of Tartu Observatory, http://www.aai.ee/~einasto.

2.   Local Dark Matter

Ernst Öpik started his studies, being a student of the Moscow University. One of the first problems he was curious about was the absorption of light in the Galaxy and the possible presence of absorbing (invisible) matter in it. He developed a method to determine the density of matter near the Galactic plane using vertical oscillations of stars. He concluded that there is no evidence for large amounts of invisible matter near the Galactic plane (Öpik 1915).

The dynamical density of matter in the Solar vicinity was investigated again by Oort (1932), who arrived at a different answer. According to his analysis the total density exceeds the density of visible stellar populations by a factor of up to 2. This limit is often called the Oort limit. This result means that the amount of invisible matter in the Solar vicinity could be approximately equal to the amount of visible matter.

The work on galactic mass modelling in Tartu Observatory was continued by Grigori Kuzmin. He developed a new method for galactic mass modelling using ellipsoids of variable density, and applied the theory to the Andromeda galaxy (Kuzmin 1943), using the recently published rotation data by Babcock (1939). Next Kuzmin turned his attention to our own Galaxy. Here the central problem was the density of matter in the Solar vicinity. The mass density can be calculated from the Poisson equation, where the dominating term is the derivative of the gravitational potential in the vertical direction. He found that this derivative can be expressed through the ratio of dispersions of velocities and coordinates in the vertical direction, $C = \sigma_z/\zeta_z$; here C is called the Kuzmin constant. Kuzmin (1952a, 1955) used data on the distribution of A and gK stars and analysed the results obtained in earlier studies by Oort (1932) and others. He obtained a weighted mean value $C = 68 \text{ km s}^{-1} \text{ kpc}^{-1}$, which leads to the density estimate $\rho = 0.08 \ M_{sun} \text{ pc}^{-3}$, in good agreement with direct density estimates of all known stellar populations (including estimates for the mass in invisible low–mass stars and white dwarfs). Two students of Kuzmin made independent analyses, using different methods and observational data (Eelsalu 1959, Jõeveer 1972, 1974) and confirmed Kuzmin results.

The local density problem was studied again by Hill (1960) and Oort (1960); both obtained considerably higher local densities of matter, and argued that there exist large amounts of dark matter in the Galactic disk. More recently Bahcall (1984) constructed a new multicomponent model of the Galaxy and determined the density of matter in the Solar vicinity, in agreement with the Oort's (1932, 1960) results. The discrepancy between various determinations of the matter density in the Solar vicinity was not solved until recently. Modern data have confirmed the results by Kuzmin and his collaborators (Gilmore, Wyse & Kuijken 1989). Thus we came to the conclusion that *there is no evidence for the presence of large amounts of dark matter in the disk of the Galaxy.* If there is some invisible matter near the galactic plane, then it is probably baryonic (low–mass stars or jupiters), since non-baryonic matter is dissipationless and cannot form a highly flattened population. Spherical distribution of the local dark matter (in quantities suggested by Oort and Bahcall) is excluded since in this case the total mass of the dark population would be very large and would influence also the rotational velocity.

3. Clusters and Groups of Galaxies

The mass discrepancy in clusters of galaxies was found by Zwicky (1933). He measured redshifts of galaxies in the Coma cluster and found that the total mass of the cluster calculated from the velocity dispersion using the virial theorem exceeds the sum of masses of visible galaxies more than tenfolds. He concluded that the cluster contains large amounts of invisible dark matter.

For some reasons the work of Zwicky escaped the attention of the astronomical community. The next step in the study of mass of systems of galaxies was made by Kahn and Woltjer (1959). They paid attention to the fact that most galaxies have positive redshifts as a result of the expansion of the Universe, only the Andromeda galaxy M31 has a negative redshift of about 120 km/s. This fact can be explained, if both galaxies, M31 and our Galaxy, form a physical system. A negative radial velocity indicates that these galaxies have already passed the apogalacticon of their relative orbit and are presently approaching each other. From the approaching velocity, mutual distance and time since passing the perigalacticon (taken equal to the present age of the Universe) the authors calculated the total mass of the double system. They found that $M_{tot} \geq 1.8 \times 10^{12} \ M_{sun}$. The conventional mass of the Galaxy and M31 is of the order of $2 \times 10^{11} \ M_{sun}$, in other words, the authors found evidence for the presence of additional mass in the Local Group of galaxies. The authors suggested that the extra mass is probably in the form of hot ionised gas; most of the paper was devoted to the analysis of the physical state of the gas. Using modern data Einasto & Lynden-Bell (1982) made a new estimate of the total mass of the Local Group, the result was $4.5 \pm 0.5 \times 10^{12} M_{sun}$ for present age of the Universe 14 Gyr. This estimate is in good agreement with new determinations of total masses of M31 and the Galaxy including their dark halos (see below).

The conventional approach for the mass determination of pairs and groups of galaxies is statistical. The method is based on the virial theorem and is almost identical to the procedure used to calculate masses of clusters of galaxies. Instead of a single pair or group a synthetic group is used consisting of a number of individual pairs or groups. These determinations yield for the mass-to-luminosity (in blue light) ratio the values $M/L_B = 1 \dots 20$ for spiral galaxy dominated pairs and $M/L_B =$

5...90 for elliptical galaxy dominated pairs (Page 1960, Burbidge & Burbidge 1961, van den Bergh 1961, Karachentsev 1976, Faber & Gallagher 1979).

The stability of clusters of galaxies was discussed in a special meeting during the IAU General Assembly (Neyman, Page & Scott 1961). Here the hypothesis of Ambartsumian on the expansion of clusters was discussed in detail. Van den Bergh (1961) drew attention to the fact that the dominating population in elliptical galaxies is the bulge consisting of old stars, indicating that cluster galaxies are old. It is very difficult to imagine how old cluster galaxies could form an instable and expanding system. These remarks did not find attention and the problem of the age and stability of clusters remained open.

4. Masses of Galaxies

Galactic Models

The classical models of spiral galaxies were constructed using rotation velocities. In contrast, the models of elliptical galaxies were found from luminosity profiles and calibrated using central velocity dispersions or motions of companion galaxies. An overview of classical methods to construct models of galaxies is given by Perek (1962).

Problems of the structure of galaxies were a major issue at the Tartu Observatory since Öpik's (1922) work on the distance of the M31, where a simple hydrostatic model of this galaxy was constructed. This work was continued by Kuzmin who developed the major principles of galactic modelling, and applied these to calculate models of M31 and the Galaxy (Kuzmin 1943, 1952b, 1953, 1956a, b). These were first models with a continuous change of the spatial density (earlier sums of ellipsoids of constant density were used). However, individual populations of galaxies were not represented in these models, in contrast to the Schmidt (1956) model of the Galaxy where different populations were included with ellipsoids of constant density. The study of kinematic and physical properties of stellar populations was made independently. For a review of the early views on the structure of galactic populations see Oort (1958), in Tartu this problem was investigated by Rootsmäe (1961).

A natural generalisation of classical and Kuzmin models was the explicit use of major stellar populations, such as the bulge, the disk, and the halo, as well as the flat population in spiral galaxies (consisting of young stars and interstellar gas). I did my PhD work on stellar kinematics in 1955 and turned thereafter my interest to galactic modelling. My goal was twofold: first, to get more accurate mass distributions in galaxies, and second, to find physical parameters of main stellar populations in both spiral and elliptical galaxies. My assumption was that similar stellar populations (say bulges) in galaxies of different morphological type should have similar physical parameters if their constituent stars have similar age and metallicity distribution. The methodical aspects of the new multicomponent models were discussed in a series of papers in Tartu Observatory Publications (in Russian with an English summary in Einasto 1969a). The spatial (or surface) density of practically all stellar populations can be expressed by a generalised exponential law (Einasto 1970b, 1974b, a similar

expression has been used independently elsewhere)

$$\rho(a) = \rho(0) \exp\left[-\left(\frac{a}{ka_0}\right)^{1/N}\right], \tag{1.1}$$

where $\rho(0) = hM/(4\pi\epsilon a_0^3)$ is the central density, $a = \sqrt{R^2 + z^2/\epsilon^2}$ is the semi-major axis of the isodensity ellipsoid, a_0 is the effective (mean) radius of the population, h and k are normalising constants, M is the mass of the population, ϵ is the axial ratio of isodensity ellipsoids, and N is a structural parameter, determining the shape of the density profile. Here we assume that isodensity ellipsoids are concentric and axially symmetric with a constant axial ratio for a given population. The case $N = 4$ corresponds to the de Vaucouleurs (1953) density law for spheroidal populations (halo), $N = 1$ corresponds to the classical exponential density law, and $N = 1/2$ to a Gaussian density law. The practical procedure of the model construction is the following. First, using photometric data for galaxies the structural parameters N of all major stellar populations are found. Next, using colorimetric and other data mass-to-luminosity ratios of populations are derived. Thereafter a preliminary mass distribution model is found and the rotation (actually circular) velocity is calculated and compared with observations. From the difference of the calculated and observed velocity corrections to model parameters are found. Initially these corrections were found using a trial-and-error procedure, later an automatic computer program was developed by our young collaborator Urmas Haud (Einasto & Haud 1989).

Mass-to-luminosity Ratios and Models of Physical Evolution of Stellar Populations

The method was applied to the Andromeda galaxy (Einasto 1969b, 1970a, Einasto & Rümmel 1970a), and to our Galaxy (Einasto 1970b). In the case of the Andromeda galaxy the mass distribution model found from the rotational data did not agree with the data on physical properties of populations. If we accepted the rotational velocities, based mostly on radio observations (Roberts 1966), then the mass-to-luminosity ratio, M/L, of central stellar populations became very low, of the order of 1 in Solar units. On the other hand, the spectral data (Spinrad 1966) suggested a much higher value, $M/L \approx 17$.

The next problem was to find internally constitent values of physical parameters of stellar populations of different age and composition. For this purpose I developed a model of physical evolution of stellar populations (Einasto 1971). When I started the modelling of physical evolution of galaxies I was not aware of similar work by Beatrice Tinsley (1968). When my work was almost finished I had the opportunity to read the PhD thesis by Beatrice. Both studies were rather similar, in some aspects my model was a bit more accurate (evolution was calculated as a continuous function of time whereas Beatrice found it for steps of 1 Gyr, also some initial parameters were different). Both models used the evolutionary tracks of stars of various composition (metallicity) and age, and the star formation rate by Salpeter (1955). I accepted a low–mass limit of star formation, $M_0 \approx 0.03 \ M_{sun}$, whereas Beatrice used a much lower mass limit to get higher mass-to-luminosity ratio for elliptical galaxies. My model yields a continuous sequence of population parameters (colour, spectral energy distribution, M/L) as a function of age. The calculated parameters of stellar populations were compared with observational data by Einasto & Kaasik (1973).

The available data supported relatively high values ($M/L \approx 10 - 30$) for old metal-rich stellar populations near centres of galaxies; moderate values ($M/L \approx 3 - 10$) for disks and bulges; and low values ($M/L \approx 1-3$) for metal-poor halo-type populations. Modern data yield slightly lower values, due to more accurate measurements of velocity dispersions in central regions of galaxies, and more accurate input data for models.

These calculations suggest that the rotation data by Roberts (1966) are biased. To find the reason for this biasing, I analysed the velocity field obtained from the radio observations. My analysis suggested that low rotational velocities in the central regions are due to a low spatial resolution of the radio beam (Einasto & Rümmel 1970b,c). The corrected velocity field was in agreement with a higher value of M/L in the central regions of M31, suggested by direct spectral data and models of physical evolution.

Mass Discrepancy on the Periphery of Galaxies

The second problem encountered in the modelling of M31 was the rotation and density distribution on the periphery. If the rotation data were taken at face value, then it was impossible to represent the rotational velocity with the sum of gravitational attractions by known stellar populations. The local value of M/L increases toward the periphery of M31 very rapidly if the mass distribution is calculated directly from the rotation velocity. All known old metal-poor halo-type stellar populations have a low $M/L \approx 1$; in contrast, on the basis of rotation data we got $M/L > 1000$ on the periphery of the galaxy, near the last point with a measured rotational velocity.

There were two possibilities to solve this controversy: to accept the presence of a new population with a very high M/L (a very uncommon property for an old stellar population), or to assume that on the periphery of galaxies there exist non-circular motions. We found that the first alternative had several serious difficulties. If the hypothetical population is of stellar origin, it must be formed much earlier than conventional populations, because all known stellar populations form a continuous sequence of kinematical and physical properties (Oort 1958, Rootsmäe 1961, Einasto 1974a), and there is no place where to include this new population in this sequence. Secondly, the star formation rate is proportional to the square of the local density (Schmidt 1959, Einasto 1972), thus stars of this population should have been formed during the contraction phase of the formation of the population near its central more dense regions (where the density is largest), and later expanded to the present distance. The only source of energy for expansion is the contraction of other stellar populations. The estimated total mass of the new population exceeded the summed mass of all previously known populations. Estimates of the energy needed for the expansion demonstrated that the mass of the new population is so large that even the contraction of all other stellar populations to a zero radius would not be sufficient to expand the new population to its present size. And, finally, it is known that star formation is not an efficient process (usually in a contracting gas cloud only about 1 % of the mass is converted to stars); thus we have a problem how to convert, in an early stage of the evolution of the Universe, a large fraction of primordial gas into this population of stars. Taking into account all these difficulties I accepted the second alternative – the presence on non-circular motions (Einasto 1969b), similar

to many other astronomers (see Materne & Tammann 1976). As I soon realised, this was a wrong decision.

Galactic Coronas

In spring 1972 I was asked to give an invited review on Galactic models at the First European Astronomy Meeting in Athens. At this time population models of galaxies had been calculated already for 5 galaxies of the Local Group and the giant elliptical galaxy M87 in the Virgo cluster. New rotation velocities suggested the presence of almost flat rotation curves on the periphery of galaxies, thus it was increasingly difficult to accept the previous concept of large non–circular motions. On the other hand, recently finished calculations of the physical evolution of stellar populations confirmed our previous view that it is extremely difficult to accept a stellar origin of the hypothetical population. My collaborator Enn Saar suggested to abandon the idea that only stellar populations exist in galaxies, to accept an idea that there may exist a population of unknown nature and origin and to look which properties it should have using available data on known stellar populations. Quickly a second set of models for galaxies was calculated, and parameters for the new dark population were found. To avoid confusion with the conventional halo population I suggested to call the new population "corona" (Einasto 1974b). The available data were insufficient to determine the outer radii and masses of coronas. Rough estimates indicated that in some galaxies the mass and radius of the corona may exceed considerably the mass and radius of all stellar populations, taken together.

To determine the parameters of galactic coronas more accurately distant test bodies are needed. After some period of thinking I realised how it is possible to check the presence of dark coronas around galaxies. If coronas are large enough, then in pairs of galaxies the companion galaxies move inside the corona, and their relative velocities can be used instead of galaxy rotation velocities to find the distribution of mass around giant galaxies. This test showed that the radii and masses of galactic coronas exceeded the radii and masses of parent galaxies by an order of magnitude! Together with A. Kaasik and E. Saar we calculated new models of galaxies including dark coronas.

In those years Soviet astronomers had the tradition to attend Caucasus Winter Schools. Our results of galactic mass modelling were reported in a Winter School in 1972. The next School was hold near the Elbrus mountain in a winter resort, in January 1974. The bottom line of my report was: *all giant galaxies have massive coronas, therefore dark matter must be the dominating component in the whole universe (at least 90 % of all matter).* In the Winter School prominent Soviet astrophysicists as Zeldovich, Shklovsky, Novikov and others participated. In the discussion after the talk two questions dominated: What is the physical nature of the dark matter? and What is its role in the evolution of the Universe? A detailed report of this study was sent to "Nature" (Einasto, Kaasik & Saar 1974).

The need for massive halos had been already suggested by Ostriker and Peebles (1973) to stabilise galaxies against bar formation. Soon after our "Nature" paper Ostriker, Peebles and Yahil (1974) published similar results using similar arguments. They used the conventional term "halo" for the dark population apparently not realising that this population cannot be of stellar origin.

Dark Matter Conferences 1975

The importance of dark matter for cosmological studies was evident, thus Tartu astronomers organised in January 1975 a conference in Tallinn devoted solely to dark matter. Historically this was the first conference on dark matter. This conference is not well known, so I give here the list of major talks:

Zeldovich: "Deuterium nucleosynthesis in the hot Universe and the density of matter";

Einasto: "Dynamical and morphological properties of galaxy systems";

Ozernoy: "The theory of galaxy formation";

Zasov: "The masses of spiral galaxies";

Fessenko: "Difficulties of the study of dynamics of galaxy systems";

Novikov: "The physical nature of galactic coronas";

Saar: "Properties of stellar halos";

Doroshkevich: "Problems of the origin of galaxies and galaxy systems";

Komberg: "Properties of the central regions of clusters of galaxies";

Vorontsov-Velyaminov: "New data on fragmenting galaxies".

As we see, the emphasis of the conference was on the discussion of the physical nature of dark matter and its role in the formation of galaxies. These preliminary studies demonstrated that both suggested models for coronas had difficulties. It is very difficult to explain the physical properties of the stellar corona, also no fast-moving stars as possible candidates for stellar coronas were found.

Stellar origin of dark matter in clusters was disussed by Napier & Guthrie (1974); they find that this is possible if the initial mass function of stars is strongly biased toward very low-mass stars. Thorstensen & Partridge (1974) discussed the suggestion made by Cameron & Truran (1971) that there may have been a pregalactic generation of stars (called now population III), all of them more massive than the Sun, which are now present as collapsed objects. They conclude that the total mass of this population is negligible, thus collapsed stars cannot make up the dark matter.

The gaseous corona of galaxies and clusters was discussed by Field (1972), Silk (1974), Tarter & Silk (1974) and Komberg & Novikov (1975). The general conclusion from these studies is that coronas of galaxies and clusters cannot consist of neutral gas (the intergalactic hot gas would ionise the coronal gas), but a corona consisting of ionised gas would be observable. Modern data show that part of the coronal matter in groups and clusters of galaxies consists of X-ray emitting hot gas, but the amount of this gas is not sufficient to explain flat rotation curves of galaxies.

The dark matter problem was discussed also during the Third European Astronomical Meeting in summer 1975. In contrast to the Tallinn Meeting now the major dispute was between the supporters of the dark matter concept and the older paradigm with conventional mass estimates of galaxies. The major arguments against the dark matter concept were summarised by Materne & Tammann (1976). They were as follows (see also Burbidge 1975):

- The dark halo hypothesis is based on the assumption that companions are physical; if they are not then they do not measure the mass of the main galaxy, but characterise mean random velocities of galaxies;

- Groups of galaxies are bound with conventional masses; the mean mass-to-luminosity ratios of groups are 4 and 30 for spiral and elliptical dominated groups, respectively;

- The high masses of clusters may be explained by the high masses of the dominant cD galaxies; in other words – there is no extra mass in clusters;

- Big Bang nucleosynthesis suggests a low-density Universe with the density parameter $\Omega \approx 0.05$; the smoothness of the Hubble flow also favours a low-density Universe.

It was clear that by sole discussion only the presence and nature of dark matter cannot be solved, new data and more detailed studies were needed.

Are Pairs of Galaxies Physical?

In mid 1970s the main arguments for the presence of dark halos (coronas) of galaxies and clusters of galaxies were statistical. In particular, the masses of double galaxies were determined by statistical methods. If companion galaxies used in mass determination are not real physical companions but random interlopers, then the mean velocity dispersion reflects random velocities of field galaxies and no conclusions on the mass distribution around giant galaxies can been made.

The difficulties connected with the statistical character of our arguments were discussed already during the Caucasus Winter School. Immediately after the school we started a study of properties of companion galaxies. The main question was: are companions true members of the satellite systems, which surround giant galaxies. Soon we discovered that companion galaxies are segregated morphologically: elliptical (non–gaseous) companions lie close to the primary galaxy whereas spiral and irregular (gaseous) companions of the same luminosity have larger distances from the primary galaxy; the distance of the segregation line from the primary galaxy depends on the luminosity of the primary galaxy (Einasto et al. 1974a). This result shows, first of all, that the companions are real members of these systems – random by-fliers cannot have such properties. Second, this result demonstrated that diffuse matter can have a certain role in the evolution of galaxy systems. The role of diffuse matter in galactic coronas was discussed in detail by Chernin, Einasto & Saar (1976). Morphological properties of companion galaxies can be explained, if we assume that at least part of the corona is gaseous. On the other hand, Komberg & Novikov (1975) demonstrated that coronas cannot be fully gaseous. Thus the nature of coronas remained unclear. Also we found that dynamical and morphological properties of primary galaxies are well correlated with properties of their companions (Einasto et al. 1976c). Brighter galaxies have companions which move with larger relative velocities than companions of fainter primaries. A further evidence of the large mass of the corona of our Galaxy came from the study of the dynamics of the Magellanic Stream (Einasto et al. 1976a).

The status of the dark matter problem in galaxies was discussed during the Commission 33 Meeting of the IAU General Assembly in Grenoble, 1976. Here arguments for the presence of dark halos and its non–stellar nature were again presented by Einasto, Jõeveer & Kaasik (1976b). But there remained two problems:

- If the massive halo (or corona) is not stellar nor gaseous, of what stuff is it made of?

- And a more general question: in Nature everything has its purpose. If 90 % of matter is dark, then this must have some purpose. What is the purpose of dark matter?

Additional Evidence for Dark Halos

In mid 1970s Vera Rubin and her collaborators developed new sensitive detectors to measure rotation curves of galaxies at very large galactocentric distances. Their results suggested that practically all spiral galaxies have extended flat rotation curves (Rubin, Ford & Thonnard 1978, 1980, see also a review by Rubin 1987). Now, for the first time, it was possible to determine the mass distribution in individual galaxies out to distances far superior to previous data. The internal mass of galaxies rised with distance almost linearly up to the last measured point (see Fig. 6 of Rubin et al. 1978). The concept of the presence of dark matter halos around galaxies was confirmed with a high confidence.

Another very important measurement was made by Faber et al. (1977). They measured the rotation velocity of the Sombrero galaxy, a S0 galaxy with a massive bulge and a very weak population of young stars and gas clouds just outside the main body of the bulge. Their data yielded for the bulge a mass-to-luminosity ratio $M/L = 3$, thus confirming our previous estimates based on less accurate data, and calculations of the physical evolution of galaxies. Velocity dispersion measurements of high accuracy also confirmed lower values of mass-to-luminosity ratios of elliptical galaxies (Faber & Jackson 1976). These results showed that the mass-to-luminosity ratios of stellar populations in spiral and elliptical galaxies are similar for a given colour (the assumption used in our model calculations), and the ratios are much lower than accepted in most earlier studies.

More recently the masses of clusters of galaxies have been determined using the temperature of hot X-ray emission gas in clusters, and by gravitational lensing. These data are discussed in other reports during this School.

By the end of 1970s most objections against the dark matter hypothesis were rejected. In particular, luminous populations of galaxies have found to have lower mass-to-luminosity ratio than expected previously, thus the presence of extra dark matter both in galaxies and clusters has been confirmed. However, the nature of dark matter and its purpose was not yet clear. Also it was not clear how to explain the Big Bang nucleosynthesis constraint on the low density of matter, and the smoothness of the Hubble flow.

5.   The Nature of Dark Matter

Neutrino-dominated Universe

Already in 1970s suggestions have been made that some sort of non-baryonic elementary particles may serve as candidates for dark matter particles. Gunn et al. (1978) considered heavy stable neutral leptons as possible candidates for dark matter particles, however in a later study Tremaine & Gunn (1979) rejected this possibility. Cowsik & McClelland (1973), Szalay & Marx (1976) and Rees (1977) noticed that neutrinos can be considered as dark matter particles; and Chernin (1981) showed that, if dark matter is non-baryonic, then this helps to explain the paradox of small temperature fluctuations of the cosmic microwave background radiation. Density perturbations of non-baryonic dark matter start growing already during the radiation-dominated era whereas the growth of baryonic matter is damped by radiation. If non-baryonic dark matter dominates dynamically, the total density perturbations can have an amplitude of the order 10^{-3} at the recombination epoch,

which is needed for the formation of the observed structure of the Universe. This problem was discussed in a conference in Tallinn in April 1981. Here all prominent Soviet cosmologists and particle physicists participated (this conference was probably the birth of the astro–particle physics). The central problem was the nature of dark matter. In the conference banquet Zeldovich hold an enthusiastic speech: *"Observers work hard in sleepless nights to collect data; theorists interpret observations, are often in error, correct their errors and try again; and there are only very rare moments of clarification. Today it is one of such rare moments when we have a holy feeling of understanding the secrets of Nature."* Non-baryonic dark matter is needed to start structure formation early enough. This example illustrates well the attitude of theorists to new observational discoveries – the Eddington's test: *"No experimental result should be believed until confirmed by theory"* (cited after Turner 2000). Now, finally, the presence of dark matter was accepted by leading theorists.

The search of dark matter can be illustrated with the words of Sherlock Holmes *"When you have eliminated the impossible, whatever remains, however improbable, must be the truth"* (cited by Binney & Tremaine 1987).

Dark Matter and the Structure of the Universe

After my talk at the Caucasus Winter School Zeldovich offered me collaboration in the study of the universe. He was developing a theory of formation of galaxies (the pancake theory, Zeldovich 1970); an alternative whirl theory was suggested by Ozernoy (1971), and a third theory of hierarchical clustering by Peebles (1971). Zeldovich asked for our help in solving the question: Can we find some observational evidence which can be used to discriminate between these theories?

Initially we had no idea how we can help Zeldovich. But soon we remembered our previous experience in the study of galactic populations: kinematical and structural properties of populations hold the memory of their previous evolution and formation (Rootsmäe 1961, Eggen, Lynden–Bell & Sandage 1962). Random velocities of galaxies are of the order of several hundred km/s, thus during the whole lifetime of the Universe galaxies have moved from their place of origin only by about $1\ h^{-1}$ Mpc (we use in this paper the Hubble constant in the units of $H_0 = 100\ h$ km s^{-1} Mpc^{-1}). In other words – if there exist some regularities in the distribution of galaxies, these regularities must reflect the conditions in the Universe during the formation of galaxies. Actually we had already some first results: the study of companion galaxies had shown that dwarf galaxies are located almost solely around giant galaxies and form together with giant galaxies systems of galaxies. In other words – the formation of galaxies occurs in larger units, not in isolation.

Thus we had a leading idea how to solve the problem of galaxy formation: *We have to study the distribution of galaxies on larger scales.* The three-dimensional distribution of galaxies, groups and clusters of galaxies can be visualised using wedge-diagrams, invented just when we started our study. My collaborator Mihkel Jõeveer prepared relatively thin wedge diagrams in sequence, and plotted in the same diagram galaxies, as well as groups and clusters of galaxies. In these diagrams regularity was clearly seen: *isolated galaxies and galaxy systems populated identical regions, and the space between these regions was empty.* This picture was quite similar to the distribution of test particles in a numerical simulation of the evolution of the structure of the Universe prepared by Doroshkevich et al. (1980) (preliminary results of this simulation were available already in 1975). In this picture a system of

high- and low-density regions was seen: high-density regions form a cellular network which surrounds large under-dense regions.

We reported our results (Jõeveer & Einasto 1978) at the IAU symposium on Large-Scale Structure of the Universe in Tallinn 1977, the first conference on this topic. The main results were: (1) galaxies, groups and clusters of galaxies are not randomly distributed but form chains, converging in superclusters; (2) the space between galaxy chains contains almost no galaxies and forms holes (voids) of diameter up to $\approx 70\ h^{-1}$ Mpc; (3) the whole picture of the distribution of galaxies and clusters resembles cells of a honeycomb, rather close to the picture predicted by Zeldovich. The presence of holes (voids) in the distribution of galaxies was reported also by other groups: Tully & Fisher (1978), Tifft & Gregory (1978), and Tarenghi et al. (1978) in the Local, Coma and Hercules superclusters, respectively. Theoretical interpretation of the observed cellular structure was discussed by Zeldovich (1978).

Our analysis gave strong support to the Zeldovich pancake scenario. This model was based essentially on the neutrino dominated dark matter model. However, some important differences between the model and observations were detected. First of all, there exists a rarefied population of test particles in voids absent in real data. This was the first indication for the presence of biasing in galaxy formation – there is primordial gas and dark matter in voids, but due to low-density no galaxy formation takes place here (Jõeveer, Einasto & Tago 1978, Einasto, Jõeveer & Saar 1980). The second difference lies in the structure of galaxy systems in high-density regions: in the model large-scale structures (superclusters) have rather diffuse forms, real superclusters consist of multiple intertwined filaments (Zeldovich, Einasto & Shandarin 1982, Oort 1983, see also Bond, Kofman & Pogosyan 1996).

Cold Dark Matter

The difficulties of the neutrino-dominated model became evident in early 1980s. A new scenario was suggested by Blumenthal, Pagels & Primack (1982), Bond, Szalay & Turner (1982), and Peebles (1982); here hypothetical particles like axions, gravitinos or photinos play the role of dark matter. Numerical simulations of structure evolution for neutrino and axion-gravitino-photino-dominated universe were made and analysed by Melott et al. (1983). All quantitative characteristics (the connectivity of the structure, the multiplicity of galaxy systems, the correlation function) of this new model fit the observational data well. This model was called subsequently the Cold Dark Matter (CDM) model, in contrast to the neutrino-based Hot Dark Matter model. Presently the CDM model with some modifications is the most accepted model of the structure evolution. The properties of the Cold Dark Matter model were analysed in detail in the classical paper by Blumenthal et al. (1984). With the acceptance of the CDM model the modern period of the study of dark matter begins.

Numerical simulations made in the framework of the Cold Dark Matter Universe (with and without the cosmological $\Lambda-$term) yield the distribution of galaxies, clusters and superclusters in good agreement with observations. These studies are too numerous to be cited here. Also the evolution of the structure can be followed by comparison of results of simulations at different epochs. During the School a movie was demonstrated showing the evolution of a central region of a supercluster (the movie was prepared at the Astrophysical Institute in Potsdam). Here the growth of a rich cluster of galaxies at the center of the supercluster could be followed. The

cluster had many merger events and has "eaten" all its nearby companions. During each merger event the cluster suffers a slight shift of its position. As merger galaxies come from all directions, the cluster sets more and more accurately to the center of the gravitational well of the supercluster. This explains the fact that very rich clusters have almost no residual motion in respect to the smooth Hubble flow. According to the old paradigm galaxies and clusters form by random hierarchical clustering and could have slow motions only in a very low-density universe (an argument against the presence of large amount of dark matter by Materne & Tammann 1976).

The amount of dark matter

In early papers on dark matter the total density due to visible and dark matter was estimated to be 0.2 of the critical cosmological density (Einasto, Kaasik & Saar 1974, Ostriker, Peebles & Yahil 1974). These estimates were based on the dynamics of galaxies in groups and clusters. In subsequent years several new independent methods were suggested. A direct method is based on the distant supernova project, which yields (for a spatially flat universe) $\Omega_m = 0.28 \pm 0.05$ (Perlmutter et al. 1998, Riess 1998). Here and below density parameters are expressed in units of the critical cosmological density. Another method is based on X-ray data on clusters of galaxies, which gives the fraction of gas in clusters, $f_{gas} = \Omega_b/\Omega_m$. If compared to the density of the baryonic matter one gets the estimate of the total density, $\Omega_m = 0.31 \pm 0.05(h/0.65)^{-1/3}$ (Mohr et al. 2000). The evolution of the cluster abundance with time also depends on the density parameter (see Bahcall et al. 1999 for a review). This method yields an estimate $\Omega_m = 0.4 \pm 0.1$ for the matter density. The formal weighted mean of these independent estimates is $\Omega_m = 0.32 \pm 0.03$. This density value is close to the value $\Omega_m = 0.3$, suggested by Ostriker & Steinhardt (1995) as a concordant model.

More recently, the density parameter has been determined from clustering in the 2-degree Field Redshift Survey (Peacock et al. 2001), and from the angular power spectrum measurements of the cosmic microwave background radiation with the Wilkinson Microwave Anisotropy Probe (WMAP) (Spergel et al. 2003). The most accurate estimates of cosmological parameters are obtained using a combined analysis of the Sloan Digital Sky Survey and the WMAP data (Tegmark et al. 2003). According to this study the matter density parameter is $\Omega_m = 0.30 \pm 0.04$. This method yields for the Hubble constant the value $h = 0.70 \pm 0.04$ independent of other direct methods. From the same dataset the authors get for the density of baryonic matter, $h^2\Omega_b = 0.0232 \pm 0.0012$, which gives $\Omega_b = 0.047$ for the above value of the Hubble constant. Comparing both density estimates we get for the dark matter density $\Omega_{DM} = \Omega_m - \Omega_b = 0.25$.

6. Summary

People often ask: Who discovered dark matter? The dark matter story is a typical scientific revolution (Kuhn 1970, Tremaine 1987). As often in a paradigm shift, there is no single discovery, the new concept was developed step-by-step.

First of all, actually there are two dark matter problems – the local dark matter close to the plane of our Galaxy, and the global dark matter surrounding galaxies and clusters of galaxies. The milestones of the local dark matter problem solution

are the studies by Öpik, Oort, Kuzmin, Bahcall and Gilmore. Dark matter in the Galactic disk, if present, must be baryonic (faint stars or jupiters). The amount of local dark matter is low, it depends on the boundary between luminous stars and faint invisible stars.

The story of the global dark matter also spans many decades. It began with the work by Zwicky (1933) on the Coma cluster, was continued with the study by Kahn and Woltjer (1959) on the dynamics of the Galaxy-M31 system, and statistical determinations of masses and mass-to-luminosity ratios of pairs, groups and clusters of galaxies. For some reason, these studies did not awake the attention of the astronomical community. However, the awareness of the presence of a controversy with masses of galaxies and galaxy systems slowly increased.

Further development of the dark matter concept was influenced by the East-West controversy during the Cold War (on this controversy see Fairall 1998, p. 11 - 12). The dark matter puzzle was solved in 1974 by two independent studies of masses of galaxies by Tartu and Princeton astronomers. It was suggested that all giant galaxies are surrounded by massive halos (coronas), and that dark matter is dynamically dominant in the Universe. As usual in scientific revolutions, the general awareness of a crisis comes when the most eminent scientists in the field begin to concentrate on the problem. This happened when the Princeton group, Burbidge (1975) and Materne & Tammann (1976) published their contributions pro and contra the dark matter hypothesis. In the following years experimenters devoted themselves to finding new evidence in favour of (or against) the new paradigm. The work by Rubin and collaborators on galaxy rotation curves, our work on properties of satellite systems of galaxies and the Magellanic stream, X-ray studies of clusters, as well as investigation of gravitational lensing in clusters belong to this type of studies.

The word on the crisis spread more rapidly in the East: the first dark matter conference was held in Tallinn in 1975, the first official IAU dark matter conference was held only ten years later. The first popular discussions of the problem were given in "Priroda" and "Zemlya i Vselennaya" (the Russian counterparts of "Scientific American" and "Sky & Telescope") by Einasto (1975) and Einasto, Chernin & Jõeveer (1975), and also in the respective journal in Estonian. In USA the first popular discussions were given many years later (Bok 1981, Rubin 1983). However, most experimental studies confirming the dark matter hypothesis were made by US astronomers, and the cold dark matter concept was also suggested by Western astronomers.

The new paradigm wins when its theoretical foundation is established. In the case of the dark matter this was done by Blumenthal et al. (1984) with the non-baryonic cold dark matter hypothesis. Also the need for non-baryonic dark matter was clarified: otherwise the main constituents of the universe – galaxies, clusters and filamentary superclusters – cannot form.

In the following years main attention was devoted to detailed elaboration of the concept of the cold dark matter dominated Universe. Here a central issue was the amount of dark matter. Initially opinions varied from a moderate density of the order of 0.2 critical density up to the critical density. Only a few years ago it was clarified that dark matter constitutes only 0.25 of the critical density, and the rest is mostly dark energy, characterized by the cosmological constant or the Ω_Λ-term.

To conclude we can say that the story of dark matter is not over yet – we still do not know of what non-baryonic particles the dark matter is made of.

Acknowledgments

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