THE CLUSTERING OF LIGHT AND OF MASS

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ABSTRACT Contradictions between cosmological theory and observations are greatly eased if luminous galaxies are not reliable tracers of the distribution of mass. However, an analysis of existing data suggests that light and mass *do* have the same distribution on scales larger than about one hundred kiloparsecs. This implies that the cosmic density parameter, $\Omega \cong 0.15$, and that baryons probably dominate the mass of the universe.

INTRODUCTION

One of the most vexing problems in cosmology today is the nature of the relative distributions of mass and light. That these might be substantially different is an idea which, virtually unknown twenty years ago, has rapidly become established as almost received truth. This remarkable transformation is due to several developments in physics and astronomy. Dynamical studies of galaxies and groups of galaxies have shown that the ratio of luminosity density to mass density is not that expected for stellar populations, and not a universal constant. Recent theories of very high energy physics have suggested the existence of forms of stable matter other than those which participate in star formation. Finally, it has become clear that theoretical attempts to understand the structure of the universe will be much more successful if the structure is rather different than that suggested by the distribution of galaxies.

The consequences for astrophysics are profound. At least 95 percent of the matter in the universe is dark. It is possible that as much as 99 percent may be dark, and composed of particles very different than the baryons which constitute the solar system and the galaxy. If the latter alternative is correct, there will also be significant consequences for the practice of astronomy. As astronomers, we are totally dependent on observations of the luminous material for all information about the distribution, motions, and properties of the matter comprising the universe. If that luminous material is a minor constituent of the universe, unrepresentative of the properties of the dominant forms of matter, the tenuous links which connect the earth-bound observer with the rest of the universe will be greatly weakened.

Regrettably, the properties of the cosmos have not been arranged for the convenience of observational astronomers, and this radical alternative may be correct, even if it is troublesome. Because the interpretation of many observations, and the confrontation of those observations with theory will be fundamentally different if this hypothesis is correct, a speedy determination of its validity is of great importance for the progress of extragalactic astronomy. In this review, I shall discuss the genesis of this idea, its consequences for astrophysics, and what existing observations may say about its correctness, with particular emphasis on scales larger than that of individual galaxies. Since masses, luminosities, and their ratio all depend on the assumed value of the Hubble Constant, H_o, I shall, where convenient, express quantities in terms of $h = H_o/100$ km s^{-1}, to make the dependence explicit. Where that is not convenient, I shall assume $H_o = 50$ km s^{-1} Mpc^{-1}.

GENESIS OF THE IDEA

Missing Mass in Galaxies

This story begins, as do many in extragalactic astronomy , with Fritz Zwicky. Until quite recently, it was universally assumed that stars were the repository of most mass in the universe. If that were so, light would naturally trace the mass, and the ratio of mass to light could be calculated from knowledge of the stellar population. In the early 1930's Zwicky (1933, 1937) applied the virial theorem to observations of the internal motions of the Virgo and Coma Clusters and deduced masses much larger than could be accounted for by the known stellar populations of the galaxies. These remarkable papers (which also discussed dark matter in clusters, tidal interactions between galaxies, and the use of gravitational lenses to measure galaxy masses) were, like many of Zwicky's, ignored for twenty years. However, by the late 1950's, the slow accumulation of additional observations had produced a body of data, on what came to be called, rather misleadingly, the "missing mass problem", which could no longer be ignored. The revival of interest in the problem led to a conference on the subject in 1961 (Neyman, Page, and Scott 1961), centered on the bizarre idea that the missing mass problem was the result of cluster explosions.

The problem remained, as a major puzzle, until the early 1970's. More and better observations only hardened the contrast between the derived masses of clusters and those of individual galaxies. The failure of conventional explanations encouraged the proliferation of exotic ones, such as those invoking non-gravitational redshifts. The breakthrough came not, as I and many others expected, from a closer examination of clusters, but rather from the discovery of large amounts of mass in the outer parts of galaxies. The most important new data were- and still remain- 21 cm observations of the outer rotation curves of spiral galaxies, which showed, unambiguously, that the mass distributions in spirals were much more extended than the light. The paper of Ostriker, Peebles, and Yahil (1974) was an important contribution, which synthesized the new results into a coherent picture of the distribution of mass in galaxies. From observations of the kinematics of galaxies and groups of galaxies, they concluded that the mass distributions about galaxies were very extended isothermal spheres, as much as 1 megaparsec in radius. The ratio of mass to light interior to a given radius appeared to increase monotonically with radius, rising from values of order unity on scales of a few kpc, to values of order 100 on scales of a megaparsec. (For an useful overview of this subject, the reader is referred to the review by Trimble, 1987 and to IAU Symposium 117 *Dark Matter in the Universe,* Kormendy and Knapp, 1987).

The Cosmological Context

Fifteen years after the paper of Ostriker, Peebles, and Yahil, it is still true that the most reliable and unambiguous determinations of dark matter only refer to the distribution of mass within the potential wells centered on individual galaxies. The progress in our understanding which they have provided is in knowing that the total masses of galaxies are greater than previously imagined: perhaps sufficiently greater to resolve the problem of missing mass raised by Zwicky. It is, of course, of great astrophysical importance to understand the nature of that dark mass, and the reason that it is distributed differently than are the visible stars. Nevertheless, the direct impact of this on the cosmological problem of the large scale structure of the universe is minimal. However, coincident with this discovery there occurred two developments in theoretical physics and astrophysics which suggested to many people that one might extend the idea of a segregation of mass and light to much larger scales.

Theoretical Difficulties

In theoretical cosmology, several decades of work on understanding the formation of structure in the universe has produced mixed results. It has proved quite easy to reproduce the qualitative distribution of galaxies, starting with a wide variety of initial conditions at the epoch of recombination, and growing structure by several processes, of which the most popular has been gravitational instability. However quantitative agreement has been almost impossible to achieve. The problems are varied. Most clustering models contain some free parameters including the shape and amplitude of the initial density fluctuation spectrum, and the value of Ω_0. The most important observational constraints are the present value of the galaxy autocorrelation function and upper limits on the temperature fluctuations within the matter at recombination, as seen in the cosmic microwave background (CMB). In no model has a combination of parameters been found which would satisfy these constraints and also produce a universe which looks like the observed one.

Voids are one problem. Even one void the size of that in Bootes (Kirshner et al. 1987) is very improbable in the volume of the universe which has been studied. In most conventional models, the probability of detecting the Bootes void with existing surveys is less than 10^{-3}. Also, voids are very large negative density fluctuations. If they have grown by gravitational instability, as most models require, their amplitudes at recombination should have been sufficiently large to perturb the CMB by much more than is observed. Even without large voids, the density fluctuations at recombination needed to produce the clustering seen today would have produced fluctuations in the CMB which are perilously close to the present upper limits.

The geometry of the clustering is another problem. It is clear that the large scale distribution of galaxies contains sheets and filaments as well as roughly spherical lumps (vid. deLapparant, Geller, and Huchra 1986). It is also rather clear that the only gravitational clustering models which can produce such features in any abundance are those in which structure formed by the fragmentation of initially smooth, very large structures, commonly called pancakes (Melott, Weinberg, and Gott 1988). Unfortunately, such models push the epoch of galaxy formation to embarrassingly recent epochs ($z = 1$, Centrella and Melott 1983) and may be inconsistent with other properties of the galaxy distribution (West, Oemler, and Dekel, 1988). There are many other quantitative difficulties with specific models. For example, the cold dark matter model cannot simultaneously reproduce the observed peculiar velocities and the two and three point correlation functions of galaxies (Davis et al. 1985). Hot dark matter models, which produce pancake structures, have too large a correlation length at the present epoch (White et al. 1983).

The conflicts between theories and observations of clustering are discouraging, but no more than that. Both the theory and the observations have a short history, and are still incomplete. Much work remains to explore a wide variety of initial conditions and possible mechanisms for growing structure. Observations of the distribution of galaxies have yet to survey a large enough volume to fairly sample the universe. It is, therefore, probably premature to conclude that conventional astrophysics is unable to account for the observed structure of the universe.

The problem of the cosmic density parameter, Ω_0, is a much more severe embarrassment. There is a very strong presumption, among cosmological theorists, that Ω_0 must be exactly unity. Part of the motivation is esthetic: $\Omega_0 = 1.0$ is the only special value, and the only stable value; any other value requires fine tuning because Ω_0 diverges rapidly from unity as the universe expands. For Ω_0 to within a factor of 10 of unity, as it clearly is, but not exactly unity, seems too much of a coincidence. The theory of inflation, an outgrowth of new ideas in particle physics (Guth 1986), also provides a physical mechanism to force Ω_0 extremely close to unity, whatever its original value.

Unfortunately, the observations stubbornly refuse to confirm this very plausible argument. The abundances of the light elements produced by primordial nucleosynthesis are only consistent with a baryon density equivalent to values of Ω_0 no greater than 0.2 (Boesgaard and Steigman 1985, Deliyannis et al. 1988). And, as I shall describe in some detail later, a variety of dynamical observations consistently give values of $\Omega_0 \cong 0.2$. Whatever may be the random and systematic uncertainties of individual estimates, the uniformly low results of a number of determinations makes a value of $\Omega_0 = 1.0$ very unlikely.

A Way Out

Simultaneous with the appearance of these frustrations in astrophysics, there occurred significant developments in theoretical physics. Much effort has been directed to unifying the three non-gravitational forces by means of a grand unified theory (GUT). A consequence of many attempts at such a theory is the existence of new types of particles, some of which are stable, of non-zero mass, and interact very weakly with ordinary baryonic matter (see, e.g. Turner 1987, Kraus 1988 for discussions of the possibilities).

A few years ago it occurred almost simultaneously to a number of people that all of the problems in theoretical cosmology enumerated above could be avoided if the universe were dominated by one of these hypothesized new forms of non-baryonic matter, distributed in a different way than the galaxies. Among the happy consequences of such a model are the following: 1- The total cosmic density can give $\Omega_0 = 1.0$ without violating the constraints on baryon density set by primordial nucleosynthesis. 2- The difficulties of growing the present structure from a universe as smooth as that seen in the CMB is removed, since the CMB is only sensitive to the baryons, which could have been more smoothly distributed than the dominant form of matter at recombination. 3- If the matter is more smoothly distributed than are the galaxies today, all dynamical estimates of mass densities and mass-to-light ratios will be systematically low. Low estimates of Ω_0 based on these quantities would, therefore, be incorrect. 4- The conflicts between clustering theories and observations could be removed, since the former predict the matter distribution, while the latter describe the galaxy distribution. 5- Since non-gravitational interactions of the baryonic and non-baryonic forms of matter are, at best, extremely weak, is should be possible to segregate them in order to form such features as the dark halos of galaxies.

Such a wonderful fix for so many theoretical problems is, obviously, too good to resist. All that is needed to make it work is (besides, of course, the actual existence of these totally speculative particles) a way of separating mass from light. Since such a separation clearly occurs in the dark halos of galaxies, the feeling has been that this is not a difficult problem. A number of mechanisms have been proposed. On small scales, such as galaxy halos, dissipation in the gas will do it. On large scales, most rely on a variable efficiency of galaxy formation to vary the ratio of mass to light. Such processes have come to be called *biased galaxy formation*, since the result is that galaxies are a biased indicator of the distribution of mass.

One of the first, and most straightforward, mechanisms of biased galaxy formation was proposed by Kaiser (1986). Suppose, as is true in many scenarios, that galaxies grow from initially small-amplitude gaussian density fluctuations, superimposed on lower amplitude, but much larger scale fluctuations. If the formation of luminous galaxies is a rare process, only occurring in $n\sigma$ peaks, where σ is the standard deviation of the density fluctuations on the scale of galaxies and $n \gg 1$, then galaxy formation will be enhanced in regions where the large scale fluctuations have maxima, and will be suppressed in large-scale minima. The result is a distribution of galaxies more strongly clustered than that of the underlying mass. This is only one possibility. Given our almost total ignorance of the process of galaxy formation, it is easy to imagine many other ways to produce a distribution of galaxies which is either more *or* less clustered than the matter. A good discussion of the possibilities has been presented by Dekel and Rees (1987).

Because the range of possible processes to separate mass from light is so large, it is very difficult to discuss, much less to test, each individually. In the remainder of this paper, I shall attempt a generic analysis, looking for properties characteristic of all, or many mechanisms. I shall try to demonstrate that, even without knowledge of the details of any particular mechanism, one can make general predictions of the observable consequences of biased galaxy formation, consequences which can, in many cases, be tested with existing observations. I shall then try to sift through observations of the properties of the galaxy population, distinguishing those which are relevant to biased galaxy formation from those which are irrelevant, and reach at least a preliminary conclusion about the viability of this idea.

MECHANISMS OF LUMINOSITY-MASS SEGREGATION

The options for segregating the luminous material from the bulk of the matter are very many. We wish to consider all of them, regardless of whether they produce a "desirable" result, such as allowing $\Omega_0 \cong 1.0$, or reconciling a particular clustering model with the observations. In Figure 1, I present a flow chart which summarizes the many choices of possible astrophysical processes. At the end points of different paths are presented summaries of possible observational and theoretical consequences of this set of choices, and a number, which is the probable maximum scale, in units of h^{-1} megaparsecs, over which these chosen processes may segregate light from mass.

The most fundamental choice is whether or not the dark matter which dominates the mass density in the universe is composed of baryons, rather than some exotic particle. We will first take the left-hand branch, and consider non-baryonic material. The next choice is whether the baryons are closely coupled to the dark matter, and trace its distribution. If they are, then for the purpose of segregation the dark matter might as well be baryons, and we move to the right-hand branch of the diagram.

If the baryons do *not* trace the dark matter, we must provide a mechanism for segregating the two components. Again, there are two choices. We may separate them after recombination, using conventional astrophysical processes, or we may invoke new physics to separate them at, or soon after, the epoch at which the particles were formed. This latter process may be a natural consequence of the physics which produced the particles, or may require additional assumptions. If such a process does operate, it is likely that the largest scale on which segregation occurs is comparable to the largest scale on which baryon (i.e. galaxy) clustering occurs, namely, about $50h^{-1}$ Mpc. Note, however, that if the baryons are *more* clumped than the dark matter at recombination, the CMB problem discussed above will be made worse, rather than better, unless the initial fluctuations were isothermal. If the baryons are *less* clumped than the dark matter, they will soon fall into the dark matter clumps, erasing the segregation.

(One conventional process, operating before recombination, can probably be ruled out. In the expanding universe, the mass contained within the event horizon grows with time. As density fluctuations corresponding to systems of a given size come within the horizon, any relativistic particles contributing to the mass density will be free to stream out of the density fluctuation. If, like photons at recombination, they are partially coupled to the baryons, they will damp out the baryon fluctuation (Silk 1974). If, however, they are very weakly coupled, the baryon fluctuation will remain, producing a universe in which the dark matter is more smoothly distributed than the baryons. Unfortunately, since the smooth dark matter dominates the mass density, baryons clumps cannot grow to form galaxies or clusters [see e.g. Bludman and Hoffman, 1986].)

Because of the many uncertainties associated with early segregation, one might choose the alternative, and assume that the baryons and dark matter had the same distribution at recombination, and any segregation occurred later. Since post-recombination physics is well understood, our options are

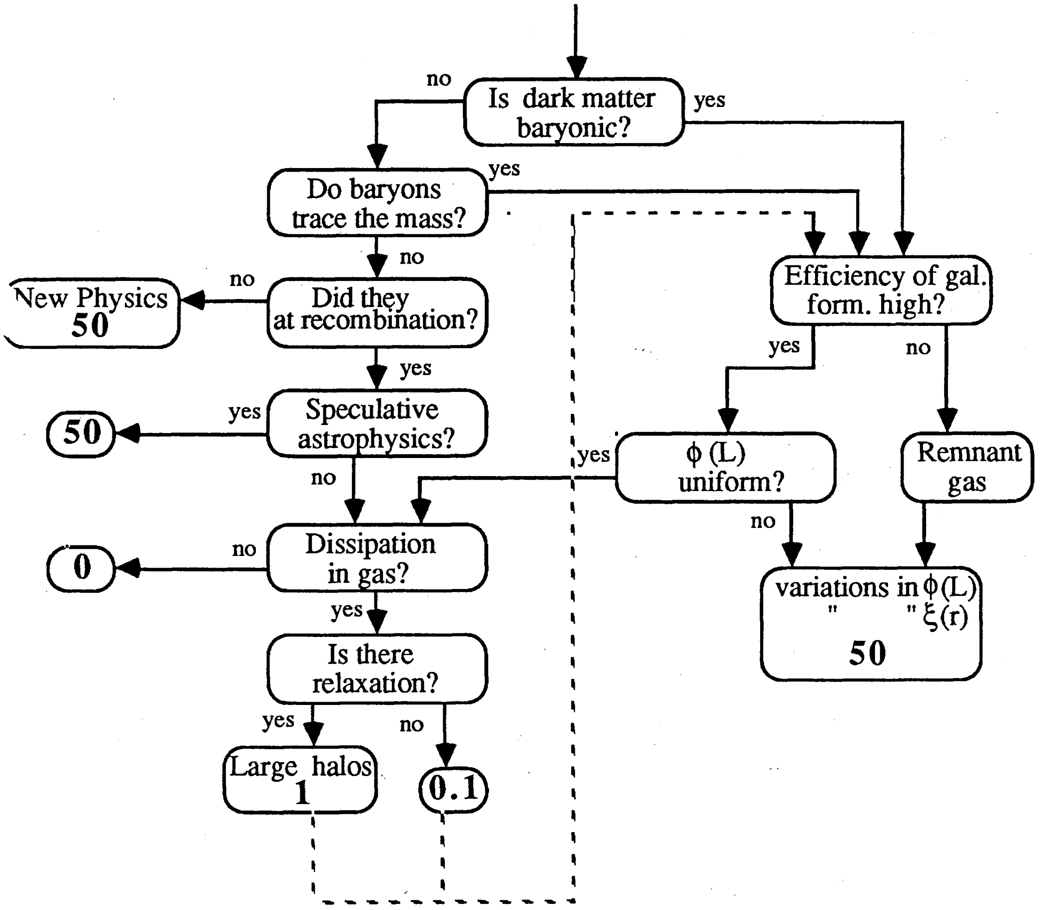

Fig. 1 Flow chart of the possible mechanisms for segregating mass from light. Numbers are maximum scale of segregation in, h^{-1} Mpc.

more limited. The only widely-discussed model which might produce a large scale segregation after recombination is the explosive galaxy formation model of Ostriker and Cowie (1981), in which blast waves from the formation of the first generation of galaxies may drive gas across large distances, separating it from the dark mater (Ostriker 1988). In the flow chart in Fig. 1, I have labeled this option "speculative astrophysics", since our knowledge of the process of galaxy formation is too incomplete to tell whether this idea is viable. Therefore, a choice must be based on taste. If one chooses this option, the maximum scale of segregation is, again, similar to that of galaxy clustering, perhaps $50h^{-1}$ Mpc.

The conservative astrophysicist, who dislikes the speculative nature of the previous mechanisms, is left with only two options. One is dissipation. Gas clouds can radiate energy, and collapse to smaller volumes; dark matter cannot. One expects, therefore, a natural segregation between the two on those scales over which radiative cooling can work in less than a Hubble time. White and Rees (1978) have argued that the maximum scale on which this can occur is that of a large galaxy. This process provides an explanation for the segregation of light and mass within galaxies, but not on scales larger than an individual galaxy halo, which, as I shall show later, seems to be about $100h^{-1}$ kpc.

If dissipation can concentrate the baryons into relatively dense lumps, one may also invoke the process of relaxation. West and Richstone (1988) have presented a model for the collapse of a cluster of galaxies from an initial configuration in which the baryons are in tight lumps (i.e. galaxies) distributed through a uniform sea of dark matter. During the phase of violent collapse, the galaxies loose energy to the dark matter. When the cluster reaches equilibrium, the process stops, but by then the galaxies are much more concentrated toward the cluster center than the dark matter. There are two conditions for such a process to work. Firstly, it can only operate in an dynamical system in which the crossing time is less than the Hubble time. The largest such systems in the universe today are clusters of galaxies, with scales of about $1h^{-1}$ Mpc. Secondly, it probably requires that the dark matter be distributed smoothly, rather than attached to individual galaxies. If it is initially attached to galaxies, in halos of, at most, $100h^{-1}$ kpc in size, it is not clear whether it will be stripped from the galaxies, and deposited in a smooth component through which the galaxies can move, rapidly enough for relaxation to work before the cluster reaches equilibrium.

The conclusion to which one comes, by following this branch of our flow diagram, is that, if one is willing to invoke speculative physics or astrophysics, one can segregate baryons and dark matter on very large scales. If one sticks to known physical processes, the two components can only be separated on scales as large as galaxies or (possibly) clusters of galaxies.

If baryons *are* the dark matter, or if we wish to segregate mass from light on larger scales than we have yet achieved, we must take the right branch of our flow diagram. With this path, the distribution of galaxies and mass can only be changed by varying the efficiency with which baryons are turned into observable galaxies. For purposes of clarity, I shall refer to these, and only these, processes as *biased galaxy formation*. The first question, therefore, is whether the efficiency of galaxy formation is uniformly high throughout the universe. Most models of biased galaxy formation assume that it is not. This implies that there are substantial amounts of remnant gas, never incorporated into galaxies, in all regions where the ratio of mass to light is higher than its minimum value.

If, on the other hand, the efficiency of galaxy formation is uniformly high, only one option remains. Almost all surveys of galaxies use samples which are limited by the apparent brightness or apparent size of the objects. Intrinsically faint or small galaxies will only appear in such a sample if they are very near, while large, luminous galaxies will be included even if very far away. Thus, studies of the distribution of galaxies are very much weighted towards large, bright objects. If one varies the luminosity function of galaxies, putting varying fractions of the baryons into faint, unobserved galaxies, the observed distribution of galaxies may differ significantly from the distribution of baryons.

The latter option has clear, and easily observable consequences. The galaxy luminosity function will vary with environment, and the galaxy correlation function will vary with luminosity. The same is likely to be true of most models in which the efficiency of galaxy formation varies, since that efficiency is unlikely to be independent of the mass, density, or depth of the potential well of a protogalaxy. One example is the application of Kaiser's mechanism to a cold dark matter clustering model which White et al. (1987) have made. Since massive galaxies come from larger, rarer density fluctuations than do low mass galaxies, their distribution is more biased. White et al. find that the autocorrelation function of galaxies with circular velocities, $v_c > 250$ km s^{-1} is higher than that of all galaxies with $v_c > 100$ km s^{-1} by a factor of about 2 to 3. The largest scale on which any of these processes of biased galaxy formation are likely to work is, again, the largest scale of galaxy clustering, about 50h^{-1} Mpc.

OBSERVATIONAL TESTS

The varied means of segregating mass from light described in the previous section have fairly clear-cut observational consequences. Firstly, *any* segregation process will, by definition, produce a mass-to-light ratio which varies over those scales on which the process operates. Dynamical mass estimates should be sensitive to this variation. Secondly, unless one invokes either pre-recombination segregation of non-baryonic dark matter from isothermal baryon lumps, or very large scale explosive galaxy formation in a non-baryon dominated universe, biased galaxy formation is the only process which can provide segregation on scales larger than a few megaparsecs. And biased galaxy formation is very likely to produce observable variations with environment in the properties of the galaxy population.

In principle, measurements of the luminosity and mass densities on a wide range of length scales can provide a complete and unambiguous answer to the question of the relative clustering of light and mass. In practice, one may not be able to determine the mass reliably on some scales without knowing the degree of relative clustering of the two components ahead of time. In that case, as we shall see, we must rely on arguments of consistency and simplicity to reach the most probable truth.

In Figure 2 I shall summarize the mass-to-light ratios obtained by the methods to be discussed below. Most derived values of M/L depend on the assumed value of H_0. Also, because the luminosity per unit *stellar* mass depends on the stellar population, which varies with Hubble type, mean values of M/L for populations of galaxies vary with the mix of morphological types in the population. Both theoretical calculations and the observed dynamics of binary galaxies suggest that the mass-to-light ratio of a population of E/S0's is about twice that of a typical spiral dominated population. Finally, because the ratio of mass to light is quoted in solar units, and the colors of galaxies are different than that of the sun, M/L, measured in solar units, depends on the photometric band. The papers quoted below have assumed various values of H_0, various photometric bands, and refer to different groups of galaxies. To minimize confusion, and put all values of M/L on a common scale, I have converted all quoted values to those appropriate for $H_0 = 50$ km s^{-1} Mpc^{-1} , for the V band, and for a population of elliptical and S0 galaxies. The reader is cautioned, therefore, that the values of $M/L)_E$ that I cite may be different than the values in the original papers. The length scales in Fig. 2 are quoted in units of h^{-1} Mpc.

By far the most reliable mass estimates are those obtained from the rotation curves of disk galaxies. The undisturbed disks of moderate and high luminosity galaxies are in circular orbit in a unique plane. Because the orbits are circular, and because the inclination of the orbital plane to the line of sight can be determined with reasonable precision, the projection of the velocities onto the line of sight can be reliably determined. Furthermore, because the orbits are circular, and the galaxies exhibit circular symmetry in the plane of the orbits, the rotation velocities are the product of a constant potential. Therefore, except for the rather small uncertainty caused by the unknown three dimensional shape of the mass distribution, the mass contained within a given radius may be obtained without ambiguity.

There now exists a large body of data, obtained from optical and 21 cm observations (vid Rubin 1987, Sancisi and van Albada 1987). One careful recent analysis, consistent with earlier work, is by Athanassoula et al. (1987). Using rotation curves and optical photometry, they construct composite models containing a stellar disk and bulge and a dark halo. Mass-to-light ratios of the stellar disks are consistent with the predictions of stellar population models, ranging from 1 for the bluest galaxies to 6 for the reddest. The stellar component dominates the central parts, but at larger radii the halo begins to dominate, causing the global mass-to-light ratio to rise by a factor of 2 to 3 by the outer edge of the galaxy. The halo mass distributions are consistent with those of unbounded isothermal spheres, out to several tens of kiloparsecs. These results are shown as the cross-hatched areas in Fig. 2. The lateral extent of these areas indicates the range of separations to which they apply, while the height is my estimate of the uncertainty in the mean. Also shown, as a smooth curve, is the run of interior mass-to-light ratio with radius in NGC 3198 (Sancisi and van Albada 1987).

Mass determinations at larger radii are dependent on other, less reliable estimators. The most commonly used has been binary galaxies, which, for most samples, measure the mass at separations of order $50h^{-1}$ kpc. The analysis of binary galaxy data is an extremely difficult problem, with a long and very controversial history. The relative velocities are small, and their measurements prone to error. The selection of an uncontaminated sample of true binary galaxies entails very large selection effects. And, the shapes of the orbits and their projection on the sky are unknown *a priori*. Together, these problems result in a potential for systematic error of at least an order of magnitude. However, in recent years considerable progress has been made in gathering reliable observations and in understanding the systematic effects. Two recent papers by White et al. (1983), and by Schweitzer (1988) illustrate the present state of the subject. Using an analytic model of the intrinsic distribution of galaxy separations, White et al. find that their data require the binary members to be on circular orbits, and to have unbounded mass

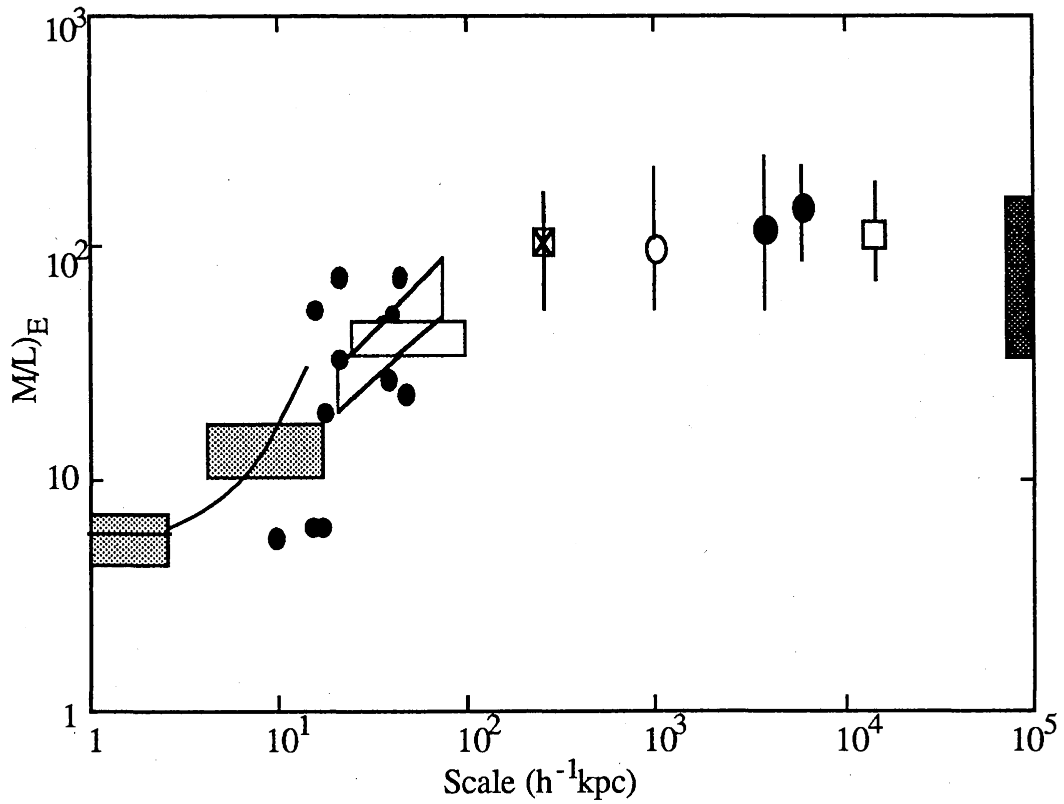

Fig. 2 Mass-to-light ratios of early-type galaxies, derived from various dynamical systems, versus the size of the system.

distributions, such that, on my scale

$$M/L = 50 \cdot (R/50h^{-1}\ \mathrm{kpc})$$

On the other hand, Schweizer (1988), using an empirically determined distribution of separations, found that her data suggested that binary members are on very elongated orbits and have bounded mass distributions, no more extended than about $35h^{-1}$ kpc The mean total mass-to-light ratio of her early-type binaries is 40. The inconsistencies between these findings are obvious. However, equally noteworthy, I think, is that the deduced M/L's are, in fact, very consistent at the scales on which they have information. This may be seen in Fig. 2, where these results are shown as unfilled boxes.

An alternate method for obtaining masses, using the hot gaseous coronae of elliptical galaxies is, in principle, much superior. If the coronae are in thermal and hydrostatic equilibrium, one may determine total mass as a function of radius, with no ambiguity, using measurements of the run of temperature and emissivity with radius. In practice existing data do not provide much information on temperature gradients, and low emissivity of the gas at large radii (emissivity is proportional to gas density squared) limita the scales over which masses can be determined. Nevertheless, fairly reliable mass estimates are available for some galaxies; the results of Forman, Jones, and Tucker (1985) are shown in Fig. 2 as small filled circles.

At somewhat larger scales, small groups of galaxies can be used. Although subject to their own systematic errors, particularly due to contamination by foreground and background galaxies, the selection effects in group dynamics are considerably less severe than in binaries. One careful analysis by Huchra and Geller (1982) uses quantitative criteria to find well-defined groups of nearby galaxies. The scatter in the M/L values of individual groups is large, because of statistical fluctuations from the very small numbers of galaxies observed within each group. However, the median value of M/L is probably reliable. On my system, $M/L)_E \cong 100$, with an estimated uncertainty of about a factor of 2. Turner et al. (1979) have handled the problem of contamination and statistical fluctuations by comparing real groups cataloged by Turner and Gott (1976) with a simulated catalog of groups constructed from N-body clustering models of the universe. The also find a mean M/L of about 100, and show that the large observed spread in the values of M/L of individual groups is, indeed, the result of sampling statistics.

Clusters of galaxies measure yet larger scales, typically several megaparsecs. The galaxy samples within rich clusters are sufficiently large that sampling fluctuations, and contamination from foreground and background objects, become unimportant. Several methods are available for estimating cluster masses. If one assumes that the radial distribution of the mass is the same as that of the galaxies, and that the distribution of velocity vectors of the galaxies is isotropic, one can calculate, as did Zwicky, the total cluster mass from the space distribution and velocity dispersion of the galaxies. If, however, one relaxes these two assumptions, a unique solution becomes more elusive. This problem has been recently analyzed by Merritt

(1987) and by The and White (1986). Both find that, with reasonably isotropic galaxy orbits, the data is most consistent with a mass distribution identical to the light distribution. However, if one allows the galaxy orbits to become progressively more circular or more elongated with increasing distance from the cluster center, it is possible to fit models in which the mass is either more or less concentrated than the galaxies. This variety of models results in total derived masses of the cluster which may be at least a factor of two greater or less than that derived if the mass distribution is the same as the galaxy distribution.

In principle, x-ray observations of the hot intracluster gas can provide very important constraints on these mass models. The gas of atoms, unlike the gas of galaxies, is dominated by collisions. Also, its kinetic energy content can be determined from its emission spectrum, with no uncertainties due to particle orbits. Therefore, if it is in hydrostatic equilibrium, its local temperature and density gradients allow us to determine the run of M(r), without prior assumptions about the overall mass distribution. Unfortunately, existing x-ray data does not provide complete temperature information, nor do observations extend to large enough distances from the cluster centers to measure the outer parts of the cluster mass. The implications of existing x-ray observations of the Coma Cluster have been discussed by Cowie, Henriksen and Mushotzky (1987) and by The and White (1988). Cowie et al. assert that the data require that the mass distribution of Coma be much more centrally concentrated than the galaxies, and the total mass be less than that usually derived from optical data. However, The and White show that a simultaneous solution using both the optical and x-ray data suggests that, while the mass is somewhat more concentrated than the galaxies in the cluster core, the two have the same distribution over most of the observed range in radii. Assuming that the mass is distributed like the galaxies, Oemler and Tucker (1988) have used optical and x-ray observations of clusters to derive a mass-to-light ratio of cluster galaxies of about 100, with an uncertainty of perhaps 30 percent. They show that the somewhat higher values found in the richest clusters is due to an unusually low efficiency of galaxy formation there. This value is shown in Fig. 2 as an open circles. The large error bar reflects the systematic uncertainty in mass distributions as well as the random errors.

Clusters of galaxies are the largest systems which have reached a steady state. All larger systems are still collapsing, and their dynamics must be analyzed using a model for the growth of clustering in the expanding universe. These models produce, directly, an estimate of Ω_o. To convert to mass-to-light ratios, I use the cosmic luminosity density, $\rho_V = 2.3\text{x}10^8 h\ L_o\ \text{Mpc}^{-3}$, derived by Kirshner et al. (1983). (That luminosity density is contributed, in large part, by spirals, so a further conversion is necessary to obtain $M/L)_E$.

The cosmic virial theorem (see e.g. Peebles 1980) relates the kinetic and potential energy content of the clustered matter in the universe. Bean et al. (1983) and Davis and Peebles (1983) have used independent estimates of the amplitude of galaxy clustering and of peculiar velocities to obtain Ω_o by this means. Bean et al. find $\Omega_o \cong 0.14$, uncertain to a factor of 2, equivalent to $M/L)_E \cong 110$. Davis and Peebles, with somewhat larger values of correlation function and peculiar velocities, find $\Omega_0 \cong 0.2$, equivalent to $M/L)_E \cong 150$, with an uncertainty of about 50 percent. The length scale to which these estimates are relevant are about the correlation length, which these authors estimate as between $4h^{-1}$ and $5h^{-1}$ Mpc. These results are presented in Fig. 2 as large filled circles.

The largest scale on which useful dynamical measurements exist is that of the local supercluster, in which the peculiar velocity of the Local Group and other outlying members of the supercluster, relative to the global Hubble expansion, provide a measure of the excess interior mass. A number of determinations of that peculiar velocity, v_p have been made. When combined with estimates for the relative overdensity in the supercluster, these yield estimates of Ω_0 which have ranged from as little as 0.04, derived from a low v_p and high overdensity (Yahil, Sandage, and Tammann 1979) to as high as 0.5, using a large v_p and small overdensity (Tonry and Davis, 1981). As best available estimates, I have used the value of $v_p = 280 \pm 60$ km s^{-1} derived by Aaronson et al. (1982) using the Tully-Fisher relation for spiral galaxies, and the overdensity, $\delta\rho/\rho = 2.0 \pm 0.2$, found by Davis and Huchra (1982) from the CFA survey. These values imply $\Omega_0 = 0.16 \pm 0.06$, or $M/L)_E = 120 \pm 43$.

What is really needed, of course, is the global value of Ω_0. If the cosmological constant, $\Lambda = 0.0$, this may be obtained from a measurement of the cosmological deceleration parameter, q_0. Although the pursuit of this number is 50 years old, we appear, alas, to be little closer to a reliable determination than we were decades ago. We may, however, determine at least that fraction of the mass density which is contributed by baryons from a measurement of the primordial abundances of the light elements D, ^{3}He, ^{4}He, and ^{7}Li, whose production in the big bang is very sensitive to density. A recent discussion of the subject by Boesgaard and Steigman (1985) suggests a lower limit, set mainly by the abundances of D and ^{3}He of $\Omega_0 h^2 > 0.01$. . New theoretical and observational work on the abundance of ^{7}Li (Deliyannis et al.1988) suggest and upper limit $\Omega_0 h^2 < 0.04$. These translate into a range of $M/L)_E$ between 32 and 130. This range is shown as the shaded area at the right edge of Fig. 2.

The straightforward interpretation of Fig. 2 is clear. Mass is more smoothly distributed than light on scales of $100h^{-1}$ kpc and less, because of the existence of heavy galactic halos. On larger scales, mass and light have a constant ratio, about 100 in solar units. Since this value is within the range allowed by primordial nucleosynthesis, the total mass density of the universe may be due to baryons. Such a picture is completely consistent with known astrophysics, which provides a natural means, dissipation of the heat content of gas by radiation, to segregate matter on scales of a galaxy, but no simple mechanism to do so on larger scales.

There are, however, other possible interpretations of these results. All existing mass determinations on scales larger than $100h^{-1}$ kpc must make the explicit assumption that light does trace matter in order to measure the mass. If the matter is more extended than the light, M/L will be underestimated. If, for example, the West and Richstone mechanism for segregation in clusters does operate, virial mass estimates will systematically underestimate total masses and mass-to-light ratios. As we have seen, the internal dynamics of clusters are entirely consistent with no segregation, but they cannot rule it out. One could, then, imagine that true values of M/L do continue to rise on scales larger than $100h^{-1}$ kpc. Obtaining an increase sufficiently large to be consistent with $\Omega_0 = 1$ will not, however, be easy. We expect that, on scales larger than about $50h^{-1}$ Mpc, light will trace mass. Unless the universe is dominated by a completely smooth, very hot component, which is difficult to arrange, we would expect the excess clumpiness of the light relative to the matter, and therefore the degree of underestimation of M/L, to smoothly decrease as the size of systems increases. For that smooth increase, from values of order 100 on scales of $100h^{-1}$ kpc to values of about 800 on scales of $50h^{-1}$ Mpc, to be completely disguised on all scales up to $12h^{-1}$ Mpc may be difficult to contrive.

Tests of Biased Galaxy Formation

Undeterred by the lack of any evidence for the existence of segregation of mass and light on large scales, we shall now look for signs, in the galaxy population, that some process of biased galaxy formation is, indeed, operating. As discussed earlier, possible evidence will be any variations with environment in the properties of galaxy populations. One such variation has been known for 50 years: although the majority of galaxies in the universe are spirals, the cores of rich clusters contain very few. One possible way of describing this phenomenon was developed by Dressler (1980), who showed that galaxy populations within clusters are correlated with the local surface density of galaxies. The fractions of E's and S0's are an increasing, and the fraction of spirals a decreasing function of density. Postman and Geller (1984) provided a significant extension of this relation. The showed that morphology was an equally sensitive function of space density, which is the more relevant physical quantity. The also showed that the dependence extended to even lower densities than Dressler had studied in clusters.

An alternate look at the same phenomenon is provided by the galaxy autocorrelation function. Davis and Geller (1976) showed that the angular correlation function of ellipticals is steeper and, at small separations, higher than that of spirals. The S0 correlation function is intermediate between those of spirals and ellipticals. Giovanelli, Haynes, and Chincarini (1986) have shown that the amplitude and radial dependence of the angular correlation function of galaxies in the vicinity of the Perseus-Pisces Supercluster steadily decreases along the Hubble sequence, and is highest for early types and lowest for latest types. Davis and Djorgovski (1985) have found a similar difference between the angular correlation functions of high and low surface brightness spirals.

These differences will not, by themselves, bias the observed distribution of galaxies: ellipticals and spirals are included almost equally in any magnitude limited sample. Might they, however, be the visible signs of some process which does bias the entire distribution? A significant clue that they are not was provided by Postman and Geller. They found that the dependence of morphology on density does not extend to very low densities. Below a density equal to about ten times the cosmic mean, morphology is independent of density. Postman and Geller noted that this density corresponded to groups in which the crossing time was equal to the Hubble time, suggesting that dynamical interactions between galaxies after formation are responsible for population differences. I had earlier reached the same conclusion, from an examination of the populations in clusters (Oemler 1974). Whatever the dependence on local density, it is also a fact that cluster populations are strongly correlated with the *global* dynamical state of the cluster (Butcher and Oemler 1979). The cores of collapsed, relaxed clusters are almost devoid of spirals, while irregular, apparently unrelaxed clusters contain many.

The conclusion that dynamical interactions, rather than biased galaxy formation, is responsible for morphological segregation is reinforced by a significant coincidence in the scale of several phenomena. Davis and Geller (1976) found that the angular correlation functions of elliptical and spiral galaxies in the UGC catalog (Nilsen 1976) were equal at a scale of about 2 degrees. There is little evidence, from their data, that they do not remain the same of larger scales. The angular correlation functions of the high and low surface brightness UGC galaxies in Davis and Djorgovski's sample (1986) are also different only on scales less than 2 degrees. The typical depth of the UGC catalog is about 4000 km s^{-1}; thus, 2 degrees corresponds to $1.3h^{-1}$ Mpc. As I shall describe later, Eder et al. (1988) have found that the correlation function of faint dwarf galaxies differs from that of giants only on scales less than $1h^{-1}$ Mpc. Now, the peculiar velocities of galaxies are about 300 km s^{-1}, almost independent of separation (Bean et. al 1983, Davis and Peebles, 1983). Therefore, a crossing time of $5h^{-1}$ billion years (expansion and recollapse in a Hubble time) corresponds (roughly) to a separation of $1.5h^{-1}$ Mpc. Thus, all observed morphological separation only occurs in environments in which interactions between galaxies can occur, which strongly suggests a causal connection. Finally, one should note that, even if one persists in ascribing these differences to biased galaxy formation, the process only works on scales of a few megaparsecs, and cannot affect the large scale distribution of galaxies.

As mentioned earlier, a more direct test of many biasing models can be derived from the relative distribution of high and low mass galaxies. Such samples are more difficult to obtain, and only a limited amount of data is available. There is no evidence that the luminosity function of galaxies varies significantly with environment. Kirshner et al. (1983) found that the luminosity function of cluster galaxies was, within the rather considerable uncertainties, the same as that of field galaxies. Phillips and Shanks (1987) have indirectly estimated the relative correlation functions of bright and faint galaxies, and have found them to be the same, but with large uncertainties. Thuan, Gott, and Schneider (1987) have shown that a sample of 58 dwarf irregulars have qualitatively the same spatial distribution as the bright galaxies in the same region volume.

Two studies of the distribution of one sample of dwarf galaxies have found significance evidence for segregation. Sharp, Jones, and Jones (1978) have looked at the angular cross-correlation of galaxies in the *Catalog of*

Galaxies and Clusters of Galaxies (Zwicky et al. 1961-1968) with a sample of dwarf irregulars studied by Fisher and Tully (1975). They find the dwarfs to be more weakly correlated with the luminous galaxies than the latter are with themselves, suggesting that they are less clustered. White, Tully, and Davis (1988) have recently analysed the space distribution of galaxies in the *Nearby Galaxies Catalog* (Tully 1988), which is, basically, a merging of the Shapley-Ames catalog of bright galaxies with Tully and Fisher's sample of dwarfs. Catalog members were subdivided into 4 groups by their internal velocities, v_i. The quantity v_i measures the depth of the galactic potential well, and is the most relevant physical quantity for many biasing mechanisms. White, Tully, and Davis find that luminous, high v_i galaxies are much more concentrated to the densest regions than are low v_i dwarfs. The degree of segregation agrees very well with that predicted by the biased cold dark matter model of White et al. (1987).

Unfortunately, as Eder et al. (1988) have shown, their are substantial problems with this sample of galaxies. As White, Tully, and Davis point out, most of the signal for a segregation comes from a few rich clusters, particularly the Virgo Cluster. Eder et al. demonstrate that the Tully sample is 90 percent incomplete for low v_i galaxies within Virgo. Overall, the incompleteness for the low velocity galaxies is about 50 percent. Therefore, the dwarf galaxies in these two studies avoid clusters because the sample does, not because of any intrinsic difference in distribution.

Because of the problems with existing samples of dwarf galaxies, we have constructed a new sample, in the vicinity of a nearby void (Eder et al. 1988). Although small- about 100 galaxies- it is well defined and homogeneous over the survey area. The objects are true dwarfs, with typical internal velocity widths of 100 km s^{-1}. We have used correlation functions and nearest neighbor distributions to compare the spatial distribution of these dwarfs with that of two samples of luminous galaxies: those in the CFA catalog, and those in the survey of UGC spirals by Giovanelli and Haynes (1985). At separations larger than $1h^{-1}$ Mpc, the distributions are identical; there is no sign of any segregation. At separations of less than $1h^{-1}$ Mpc, the lower velocity width dwarfs, those with $v_i < 100$ km s^{-1}, are underabundant by a factor of about 10. As mentioned earlier, I suspect that this depletion is the result of the same processes of galaxy-galaxy interactions which is responsible for the morphology-density relation among galaxies.

DISCUSSION

The work which has been done so far represents only a first essay at the problem of detecting the signs of biased galaxy formation. Only a limited range of possible effects have been studied, and in only a small volume of the universe. It is much too early to pronounce final judgement on the viability of this idea. Nevertheless, it is true that, at the present time, there is *no* observational evidence suggesting the existence of galaxy biasing, and there is some evidence against its occurrence. Similarly, the determinations of mass-to-light ratios on a variety of scales leave much to be desired. Many of the dynamical tests produce ambiguous results. Many depend on the assumption that light does trace mass, and their results may be systematically too low if it does not. Nevertheless, the data, taken at face value, suggest that mass and light have a constant ratio on scales larger than an individual galactic halo, and provide no support for the contrary view.

What is most striking about these many findings is their consistency. An outside observer, studying the observational data in blissful ignorance of cosmological theory, would, I think, be astonished by the suggestion that the universe is dominated by some exotic form of dark matter with a very different distribution than that of the galaxies. The products of primordial nucleosysnthesis, the observed dynamics of galactic systems, and the properties of the galaxy population all imply that the universe is dominated by baryons, of density sufficient to give $\Omega_o \cong 0.15$, and the baryons reside in galaxies and their halos.

One might object that the idea of rather smoothly distributed non-baryonic dark matter is not an arbitrary fudge, because the dark halos of galaxies show that at least some exists. However, this need not be so. Since, as Fig. 2 shows, the mass density in dark halos is consistent with the mass density in baryons, there is no particular need to invoke other forms of matter. In fact, if galaxy formation is efficient, so that most baryons are incorporated into galaxies, then at least some fraction of the dark halos must be baryonic. The luminous matter in galaxies, whose $M/L \cong 4$, provides a cosmic mass density equivalent to $\Omega_o = 0.007$. Primordial nucleosynthesis suggests that $\Omega_o h^2 > 0.01$. Thus, unless $H_o > 120$ km s^{-1} Mpc^{-1} , there are more baryons in the universe than can be accounted for by the luminous parts of galaxies. Choices of H_o are a matter of taste. If one believes, as I do, that the ages of the globular clusters requires that $H_o \cong 50$ km s^{-1} Mpc^{-1}, then the total baryonic mass to light ratio of the universe, $M/L)_b > 23$. The dark halos are the logical repository of these extra baryons. Oemler and Tucker (1988) have shown that the mass-to-light ratios and gas contents of clusters of galaxies are most consistent with a baryonic mass-to-light ratio of about 100. If we are correct, then the dark halos are entirely baryonic.

The universe is a complicated place, and things are not always what they appear. Nevertheless, Occam's razor remains a useful guide. To abandon the straightforward interpretation of observations for a much more convoluted explanation requires a very compelling justification. It is not clear that the current difficulties in understanding the universe are sufficiently severe to provide that. We do not, as yet, understand physics at the GUT scale and beyond, nor do we understand the early universe, or the process of galaxy formation. It is hardly surprising that current theory cannot explain everything.

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EXERCISE

The fraction of diffuse light and stars in clusters is proportional to the cluster luminosity, L. We would like to explain this material as the remains of protogalaxies, which were disrupted by collisions. We have established that the cluster mass,

$$M \sim L^{5/4}$$

and the cluster size

$$R \sim L^{1/2}$$

a) Assuming virial equilibrium, find the rms velocity of cluster galaxies v, as a function of M. Show that the collision rate does not scale properly with L. Estimate its value in years^{-1}.

b) Assume all clouds have the same mass, radius, and velocity, and assume that when clouds collide, all of their kinetic energy goes into heat. Assume clouds, before collision, are in virial equilibrium with mass M_c and radius r_c, and that disruption occurs if after collision, $-E_{grav} < E_{thermal}$. How does the disruption rate change with cluster mass?

c) Recalculate b, taking account of the fact that the distribution of galaxy velocities in a cluster goes like

$$N(v) \sim e^{-v^2/2\sigma^2} \text{ in one dimension.}$$

d) Suppose that, whatever E_{grav}/E_{therm}, a cloud can survive a collision if it can cool down before the next collision. Suppose the cooling rate

$$\frac{dE_{therm}}{dt} \sim T.$$

Recalculate (c).