The Modified Dynamics–A Status Review

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Abstract. The Modified dynamics (MOND) has been propounded as an alternative to Dark matter. It imputes the mass discrepancy in galaxy systems to failure of standard dynamics in the limit of small accelerations. After a brief description of the MOND tenets, I discuss how its predictions now compare with the data. I put special emphasis on rotation-curve analysis–whence comes the most clear-cut support for MOND, and on the cores of rich x-ray clusters, where MOND does not yet explain away the mass discrepancy. I then outline the MOND program, especially work still left to do. This is followed by general comments on cosmology and structure formation in MOND. I conclude with some incomplete thoughts on the possible origin of MOND (as an effective theory); in particular on the possibility that it comes about as a vacuum effect.

1. Introduction–the basic tenets of MOND

To speak of the "dark matter" problem is to beg one of the most important conundrums in present-day science; after all we have no direct evidence that dark matter actually exists in appreciable quantities. All we know is that the masses directly observed in galactic systems fall below what is calculated using standard dynamics. Stuffing galactic systems and the universe with putative dark matter is perhaps the least painful remedy for most people, but it is not the only one possible. Another avenue worthy of consideration builds on a possible failure of standard dynamics under the conditions that prevail in galactic systems. As you may know, the modified

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dynamics (MOND) has been put forth in just this vein[1]. It hinges on the accelerations in galactic systems being very small compared with what is encountered in the solar system, say. MOND asserts that non-relativistic dynamics involves the constant a_0, with the dimensions of acceleration, so that in the formal limit $a_0 \rightarrow 0$–i.e., when all quantities with the dimensions of acceleration are much larger than a_0–standard dynamics obtains (in analogy with the appearance of $\hbar$ in quantum mechanics, and the classical limit for $\hbar \rightarrow 0$). In the opposite (MOND) limit of large a_0 dynamics is marked by reduced inertia; one may roughly say that in this limit inertia at acceleration a is ma^2/a_0, instead of the standard ma. This still allows for different specific formulations. Indeed we have nonrelativistic formulations of MOND, derivable from actions, based on either modified gravity[2], or on modified inertia[3]; these will be described below. A simple, if primitive, formulation that captures much of the content of MOND, and which gives the basic idea, is this: Imagine a test particle in the gravitational field of some mass distribution whose standard (Newtonian) gravitational acceleration field is $\mathbf{g}_{\rm N}$. In standard dynamics the acceleration, $\mathbf{g}$, of the particle is $\mathbf{g}_{\rm N}$ itself. MOND posits that this is so only in the limit $\mathrm{g}_{\rm N} \gg a_0$. In the opposite limit $\mathrm{g}_{\rm N} \ll a_0$ we have roughly $\mathrm{g} \sim (\mathrm{g}_{\rm N} a_0)^{1/2}$. To interpolate between the limits we use a relation of the form $\mu(\mathrm{g}/a_0)\mathbf{g} = \mathbf{g}_{\rm N}$, where $\mu(x) \approx x$ for $x \ll 1$, and $\mu(x) \approx 1$ when $x \gg 1$. This relation gives an approximate relation between the typical accelerations in a system (as embodied, say, in an exact virial relation derived from an exact theory). It also gives a very good approximation for the acceleration in circular motion relevant for rotation curves of disc galaxies[3][4] (in modified inertia theories it give the exact rotation curve). In the more decent formulations of MOND, the actual acceleration of a test particle is not directly related to the local Newtonian acceleration as in the above relation (in particular, the two are not in the same direction, in general).

Some immediate, and unavoidable, predictions of even the basic tenets are[1][5]:

1. The rotation curve for any isolated body becomes flat, asymptotically.

2. The asymptotic rotational velocity, V_∞, depends only on the total mass of the body, M, via $V_\infty^4 = MGa_0$. This predicts a Tully-Fisher relation between velocity and luminosity if the M/L values are narrowly distributed.

3. A similar approximate relation exists, for a body supported by random motions, between the mean velocity dispersion and the total mass. This is relevant to mass determinations of systems such as dwarf-spheroidal, and elliptical, galaxies, and of galaxy groups and clusters. It also predicts an approximate $L \propto \sigma^4$ relation in such systems (with similar M/L values).

4. The smaller the typical acceleration of a gravitationally bound sys-

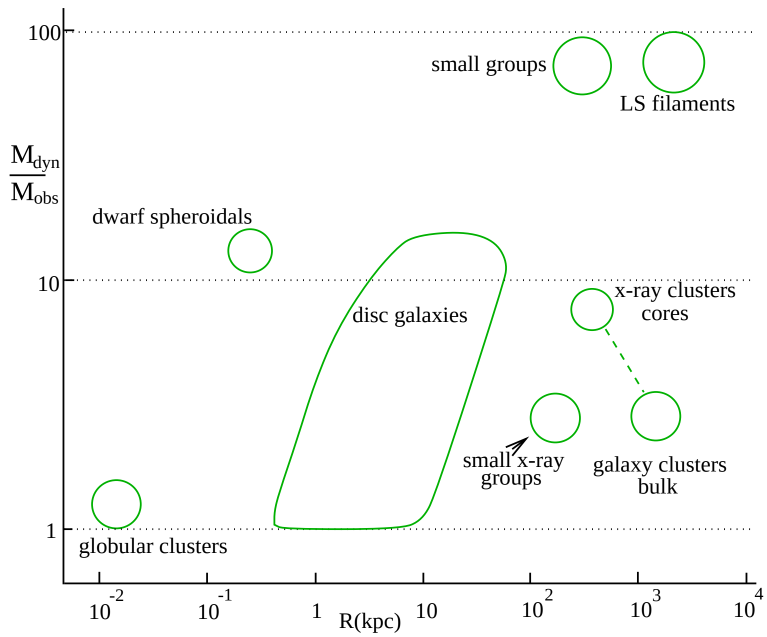

Figure 1. The mass discrepancy (dynamical mass over detected mass) in various galactic systems plotted against the typical system size.

tem, the larger the mass discrepancy it should evince. It had thus been predicted that all low-surface-brightness (LSB) systems should evince large mass discrepancies since, for a given M/L, surface brightness is proportional to acceleration (in the mean). This pertains, e.g., to dwarf-spheroidal satellites of the Milky Way, and to low-surface-brightness disc galaxies.

5. Above all, the full rotation curve of a disc galaxy should be obtained, using MOND, from the distribution of the observed mass alone.

Comparison with the data, as discussed later, yields a value of a_0, determined in several, independent ways (using the different roles of a_0 in the theory). Very interestingly, the value a_0 turns out to be of the same order as cH_0–an acceleration parameter of cosmological significance[1]. Anticipating later discussion, I remark here that this might be a crucial clue as to the origin of MOND, and its possible origin in effects related to cosmology.

2. The performance of MOND

Figure 1 summarizes the mass discrepancy in various galactic systems. It shows, approximately, the ratio of the dynamical mass, as determine with

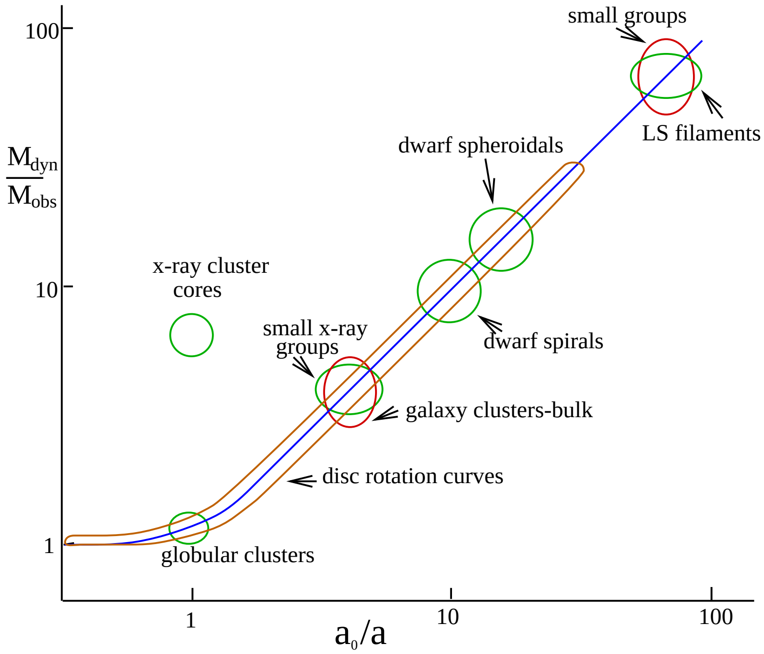

Figure 2. The mass discrepancy plotted against the typical system acceleration.

standard dynamics, to the mass so far accounted for by direct observations. The discrepancy is plotted against some "typical" system radius. (Masses in galactic systems show no sign of saturation with radius, and the value within that "typical" radius is used.) I note, in passing, that there is no correlation of the discrepancy with system size. Remark, in particular, that the small dwarf spheroidals and LSB discs show large discrepancies, while the large galaxy clusters evince only moderate discrepancies. This flies in the face of attempts to explain away the mass discrepancy by modifying gravity at large distances, predicting increase in the "discrepancy" with size. (Contrary to some lingering misconception, MOND is not a modification at large distances, but at low accelerations–which for a given mass are attained at large distances.)

The use of MOND dynamics should eliminate the mass discrepancy in all systems. Put differently, MOND predicts the mass discrepancy expected when using Newtonian dynamics. Figure 2 shows the discrepancy plotted now against the typical inverse acceleration–as prescribed by MOND. It also shows the MOND prediction of the discrepancy as a solid line interpolating the value 1 at low a^{-1} and the predicted discrepancy, a_0/a, at $a \ll a_0$.

The positions of the blobs describing the different galactic systems roughly represent detailed work on individual systems: dwarf spheroidals[6]-[9], disc-galaxy rotation curves[10]-[13], galaxy groups[14], x-ray clusters (e.g. [15]-[18]), and large-scale filaments[19]. I expand here on two types of systems.

2.1. Cores of rich x-ray clusters

We see in Fig. 2 that MOND explains away the mass discrepancy in all the systems studied except one–the cores of rich x-ray clusters. This is discussed in [20], but the point had been made before in ref.[16] (and a hint of it is in ref.[15], see also ref.[18]). (I separate, somewhat artificially, the results for cluster cores, within a few hundred kiloparsecs, from the cluster bulk within a few megaparsecs. There is, of course, continuity: the discrepancy in the core, which still lingers in MOND, decreases and disappears as we go to larger radii.) The point is that x-ray-cluster cores have, by and large, borderline accelerations (i.e. of order a_0 or somewhat larger). MOND tells us then not to expect much of a mass discrepancy there, when, in fact, the mass so far accounted for (in hot gas, and stars) falls short of the dynamical mass obtained from gas hydrostatics, and from strong lensing. According to MOND there must then reside in these cores normal baryonic matter yet undiscovered. It is well known that such clusters are characterized by cooling flows that deposit large quantities of matter in their cores. These deposits have not yet been discovered, and, it is surmised, might be in the form of dim stars or warm gas. Present-day mass-deposition rates do not suffice to supply the required mass within the Hubble time, but the rates might have been higher in the past. In any event it is a strong prediction of MOND that the dark matter in cluster cores is baryonic and will be detected. The recent detection of strong UV emission from the cluster Abell 1795 has been interpreted as arising from warm gas enough to account for the dark matter in the core[21].

From an historical perspective, it is interesting to remember that at the time of the advent of MOND it was not known that clusters harbor large quantities of hot, x-ray-emitting gas. This, as we now know, constitutes the lion's share of the baryonic mass in x-ray clusters. Similar to the case with the cluster cores now, the MOND analysis of the time[22] still left a mass discrepancy for some clusters (such as Coma, A2029, A2199, A2256). Seeing that these clusters are x-ray sources, it had been surmised[22] that intergalactic gas responsible for the emission might account for the lingering discrepancy, as indeed proved to be the case.

2.2. Rotation-curve analysis

Rotation-curve analysis is arguably the heart of MOND testing. It surpasses all other tests as regards the quality of the data, the freedom from astrophysical assumptions, and the range of acceleration values covered. About eighty disc galaxies with sufficient data (extended, two-dimensional velocity maps, photometry, and HI distribution) have now been successfully MOND tested by various studies[23][10][24][11][12][13]. For each galaxy the analysis involves, in most cases, one adjustable parameter–the M/L value of the stellar disc (in standard dark-halo fits there are two additional free parameters characterizing the halo). A success of MOND for even a single rotation curve is most significant, because even full freedom to choose M/L is anything but sufficient to make MOND work in any given case. This is nicely demonstrated in ref [13] by analyzing a synthetic galaxy model taken to have the HI data (HI distribution, and rotation curve) from one galaxy, and the stellar light distribution from another. An attempt to fit this galaxy with MOND gives a very bad best fit, and the best-fit M/L value is unreasonably high. In contrast, a standard, dark-halo fit for this "wrong" galaxy model gives a very good fit (with reasonable M/L value). Another example serving to demonstrate the limited leverage of the M/L parameter: Many galaxies have high accelerations ($a \gg a_0$) in their inner parts; MOND then predicts no discrepancy there, and the stellar M/L value is thus fixed by the inner parts. The rotation curve in the outer parts (its shape, whether falling or rising, and amplitude) then remains an unadjustable prediction of MOND that could easily fail.

In addition, note that the stellar M/L value is not really a totally free parameter. In must fall within some acceptable range, and, by and large, is constrained by theoretical models. The study of ref.[12], which is unique in its use of the infrared K' photometric band–arguably the best representative of stellar mass–shows that, indeed, the resulting MOND M/L values, for the sample of Ursa Major galaxies studied, are very narrowly distributed near one solar unit. The study also finds that the B-band, MOND M/L values are strongly correlated with the observed galaxy color, following the expected theoretical relation. All this shows M/L to be a rather tightly tethered parameter, which further heightens the significance of the successful MOND analysis.

3. The general MOND program

Those cleaving to Newtonian dynamics may take the success of MOND to reflect some very strict regularity–encompassing the whole gamut of galactic systems–relating the distribution of visible matter to that of dark matter via a simple formula. The few of us who have contributed to MOND

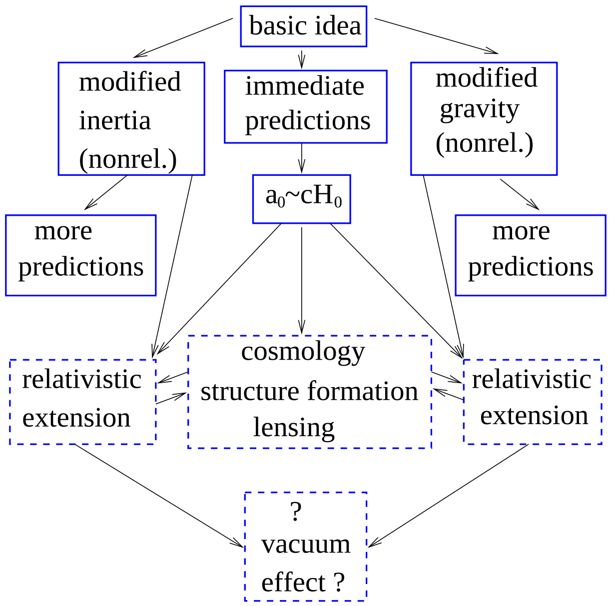

Figure 3. Schematics of the MOND program.

theory and testing over the years view this success as strong indication of departure from standard dynamics in the parameter region relevant to galactic systems. Taking MOND in such a vein, one seeks to construct theories, with increasing depth and compass, that incorporate the basic tenets of MOND. Figure 3 presents the schematics of these efforts, with full-line blocks marking areas in advanced stages of development.

At the nonrelativistic level, at least, MOND may be viewed as either a modification of gravity, or a modification of inertia[1]. In the former, the gravitational field produced by a given mass distribution is dictated by a new equation; in the latter the equation of motion is MONDified, while the force fields remain intact. An example of the former is the MONDification of the Poisson equation discussed in ref. [2] where the gravitational potential, φ, is determined by the mass distribution, ρ, via

$$\vec{\nabla}\cdot[\mu(|\vec{\nabla}\varphi|/a_0)\vec{\nabla}\varphi] = 4\pi G\rho. \tag{1}$$

Mondified inertia is discussed in refs. [3][25]. In such theories, when derived from an action, one replaces the standard kinetic action for a particle ($\int v^2/2\ dt$) by a kinetic action that is a more complicated functional of the particle trajectory

$$A_m S[\mathbf{r}(t), a_0], \tag{2}$$

where A_m depends only on the body, and can be identified with its mass, and S depends only on the trajectory and on a_0 as a parameter. Weak equivalence is thus insured. In the formal limit $a_0 \rightarrow 0$ the action goes to the standard kinetic action. In the opposite limit, $a_0 \rightarrow \infty$, $S \propto a_0^{-1}$, and inertia disappears in the very limit.

With respect to Newtonian dynamics, special relativistic dynamics is an example of modified inertia: The equation of motion of a relativistic particle moving in a force field $\mathbf{F}(\mathbf{r})$ is $md(\gamma\mathbf{v})/dt = m\gamma[\mathbf{a} + \gamma^2 c^{-2}(\mathbf{v}\cdot\mathbf{a})\mathbf{v}] = \mathbf{F}(\mathbf{r})$, derived from the kinetic action $mc^2 \int d\tau = mc^2 \int \gamma^{-1}\ dt$. Here too there appears a parameter, c, which, like a_0 in MOND (and $\hbar$ in QM) both delimits the standard (classical) region, and enters the dynamics in the non-classical regime. Unlike the special-relativistic action, which is still local, the MOND action is perforce non-local if it is to be Galilei invariant[3].

Mondified gravity and mondified inertia do not differ on what we call the basic predictions of MOND: The asymptotic flatness of rotation curves (and their general shape), the $M \propto V^4$ relation, the added stability of systems in the deep MOND regime[26], etc. There are, however, important differences; some examples are: 1. In mondified gravity only systems governed by pure gravity (such as galactic systems) are affected, while in mondified inertia the modification applies for whatever combination of forces is at play. 2. in the former, the acceleration of a test particle depends only on its position in the field, while in the latter it depends strongly on other details of the trajectory (inertia is identified with acceleration only in standard Newtonian dynamics). As an example, we can see in the special-relativity case, mentioned above, that the $\mathbf{v}\cdot\mathbf{a}$ term vanishes for a circular orbit, but dominates, at high γ, for a linear trajectory. 3. In mondified inertia the expressions for the conserved quantities and adiabatic invariants in terms of the motion are modified[3], in contradistinction to mondified gravity.

An acceptable relativistic extension for MOND is not yet at hand. Discussions of various candidates can be found in refs. [2][27][28][29], but each of these has its problems. These problems seem to be specific to the particular models (e.g. that in [2] has superluminal modes, scalar-tensor theories as discussed in [28] do not give as large a light bending as is observed, and that in [29] is based on a non-dynamical pregeometry).

Reflection over this question has convinced me that a relativistic extension will not just be a relativistic theory where a_0 appears as a parameter, with GR restored in the limit $a_0 \rightarrow 0$. I have always viewed MOND as an effective theory (i.e. an approximate theory that results from a deeper one in a certain limit, and/or when some of the relevant degrees of freedom are integrated out). In the present case MOND is perhaps an approximation in the limit of small sizes and short times (on the cosmological scale), and nonrelativistic motion, due to some yet-undiscovered effect connected with cosmology. An analogy will highlight the point: If we are ignorant of earth gravity as derived from the pull of the earth–such as when we are immured forever in a small laboratory near the earth's surface–dynamics is described approximately by modified inertia of the form

$$\mathbf{F} = m(\mathbf{a} - \mathbf{g}), \tag{3}$$

where $\mathbf{F}$ is the applied force excluding earth gravity, and $\mathbf{g}$ is the free-fall acceleration on earth. This can be recast to resemble MOND inertia:

$$\mathbf{F} = m\overleftrightarrow{\mu}(\mathbf{a}/\mathrm{g}) \cdot \mathbf{a}, \tag{4}$$

where $\overleftrightarrow{\mu}(\mathbf{x}) \equiv 1 - \frac{\mathbf{e}\otimes\mathbf{x}}{\mathrm{x}^2}$, and $\mathbf{e} \equiv \mathbf{g}/\mathrm{g}$ is a down-pointing unit vector. This is a good approximation inasmuch as this proverbial laboratory is our whole universe; i.e., for systems small compared with $R_\oplus$ (analogous to the Hubble distance), and times small compared with $H_\oplus^{-1} = t_\oplus \equiv R_\oplus/c_\oplus$, where $c_\oplus = (M_\oplus G/R_\oplus)^{1/2}$ is the escape speed–analogous to the speed of light. The effective "acceleration constant", g, appearing in this modified inertia is related to the "cosmological" parameters by $\mathrm{g} = c_\oplus H_\oplus$.

In a relativistic extension of MOND, or in the cosmological context, a_0 may lose its role as a "universal constant"[3][25]–as g does in the above analogy when dealing with, say, satellite motion for which $v \sim c_\oplus$. The peculiar situation is further highlighted by the fact that–in view of $a_0 \sim cH_0$–the only system that is both high-field in the GR sense and in the deep-MOND regime is the universe at large. (In the quantum analogue a system in the high-field, quantum regime is of Planck scale or smaller. There, we can, at least look from outside the Planck scale, which we cannot do in MOND.) Relativistic MOND must then be understood as part and parcel of cosmology, as I elaborate more in the next section.

4. Cosmology and structure formation

Cosmology is then not simply an application of a relativistic version of MOND but a unit with it. The key to finding the underlying theory may lie in understanding first how an acceleration of cosmological significance can, at all, enter local dynamics, which I discuss in the last section.

If a_0 is a fingerprint of cosmology on local dynamics, it is not necessarily the identification $a_0 \sim a_{ex} \equiv cH_0$ which is the the right one. There are other cosmological acceleration scales[30][3][31] such as $a_c \equiv c^2/R_c$, where R_c is the curvature radius (spatial or space-time), or $a_\Lambda \equiv c\Lambda^{1/2}$, where Λ is the cosmological constant. Today we have only upper limits on a_c, which is of the order of a_{ex}. Several pieces of evidence seem now to imply a non-zero cosmological-constant with $\Lambda \sim H_0^2$. If this is true then we also have $a_0 \sim a_\Lambda$. Thus a_0 might be a proxy for any of the cosmological acceleration parameters. Since these depend differently on cosmic time, a_0 may vary with cosmic time in a way that is difficult to know without the correct identification. Such possible variation of a_0 has obvious ramifications for the formation and the ensuing evolution of galactic systems.

Even without a theory we can make out some semi-quantitative aspects by which MOND cosmology must differ greatly from standard cosmology:

1. MOND is based on the phenomenology of galactic systems and hence, in principle, is not committed on the question of cosmologically homogeneous component of dark matter. But certainly, it is in the spirit of MOND that we should not conjecture the existence of any DM component without first trying to explain it away with new physics. Recent leanings toward a non-zero cosmological constant(CC) are a step in this direction. And perhaps[25] the same mechanism that produces a CC-like contribution might also effect MOND (hence the coincidence $a_0 \sim c\Lambda^{1/2}$). At any event, a MOND-inspired cosmology would start with no dark matter.

2. The MOND Jeans mass–a basic concept in structure formation, which indicates which masses are likely to collapse from an homogeneous medium–depends differently on the temperature, T, and density, ρ, of the medium[30]: $M_J(\mathrm{MOND}) \propto T^2/a_0$, instead of the Newtonian dependence $M_J \propto T^{3/2}\rho^{-1/2}$.

3. The acceleration in a collapsing system increases as the collapse proceeds (after detachment from the Hubble flow). If a_0 varies at all, it is expected to decrease with cosmic time. So, the effect of MOND is expected to decrease with time in a collapsing system. (The system would behave as if the fraction of fictitious dark matter it harbors decreases with time.)

In default of a theory one can still attempt to obtain approximate MOND cosmologies–in order to get a hint of what is expected–by supplementing nonrelativistic MOND with extra assumptions. For instance, one might assume that a_0 does not vary with cosmic time, identifying it with a veritable cosmological constant[30]. This is done in ref. [32] where some further tentative assumptions are made. In such a case one is bound to ask why it is that this constant a_0 is today of the same order as the variable cH_0. The same question arises in connection with the emerging value of the cosmological constant $\Lambda \sim H_0^2$. In MOND, at any rate, this could find an antropic explanation whereby structure formation (hence star formation and the eventual development of mankind) is facilitated when the acceleration within the horizon ($\sim cH_0$)–decreasing as it does with cosmic time–becomes similar to the crucial dynamical constant a_0[30][32].

5. A possible origin of MOND

Why should then a cosmological acceleration parameter enter local dynamics in galaxy systems? I have discussed this question in refs.[3][25] and give here a brief account. I shall concentrate on mondified inertia, which seems to me more promising at this juncture.

The thread I would like to follow is that inertia might result from the interaction of matter with the vacuum. Also, cosmology affects the vacuum and is affected by it (e.g. through a contribution to a cosmological constant). So, either cosmology affects inertia through the intermediary vacuum, or, cosmology and inertia are both affected by the vacuum dynamics, which then enters cosmology, say, as a cosmological constant, Λ, and MOND through $a_0 \approx c\Lambda^{1/2}$.

Inertia is what makes kinematics into dynamics, associating with motion the attributes of energy and momentum that can be changed only by applying forces, as described by the appropriate equation of motion. Just how much energy and momentum is associated with so much motion is dictated by the kinetic action of the relevant degrees of freedom. To obtain inertia as a derived effect is to derive the kinetic actions (in our case from some vacuum effect). From this action the energy-momentum tensor is derived; thus, in relativity, this action also encapsules the contribution of the particular degree of freedom to the sources of gravity. Attempts to derive inertia–in the spirit of Mach's principle–have concentrated mainly on inertia of bodies–see e.g. ref. [33]. But, of course, all dynamical degrees of freedom, whether we describe them as bodies (particles) or fields, carry inertia.

Supposedly one starts from only interactions between the different degrees of freedom and get inertia in the form of effective kinetic actions. We know that interactions can, indeed, induce and modify inertial actions. For example, the effective mass of "free" electrons and holes in a semiconductor can be greatly changed from its vacuum value; mass renormalization in field theory is, of course, a vacuum effect; and the Higgs mechanism induces an effective mass term from the interaction with the putative Higgs field. It is also known that the interaction of the electromagnetic field with charged vacuum fields begets a free effective action for the electromagnetic field–the Heisenberg-Euler effective action (see e.g. [34] and [35] p. 195). What role, if any, these mechanisms play in MOND is not clear. However, since they are known to affect inertia, they must be reckoned with in any complete analysis.

The scheme I have in mind is inspired by Sakharov's proposal[36] to derive the "free" (Einstein-Hilbert) action of gravity from effects of the vacuum: Curvature of space-time modifies the dynamical behavior of vacuum fields, hence producing an associated energy or action for the metric field. To lowest order (in the Planck length over the curvature radius) this gives the desired expression $\int g^{1/2}R$. Sakharov's arguments make use of the fact that the vacuum fields have inertia (since they are assumed to carry the usual energy-momentum). So, derived inertia comes prior to induced gravity a-la Sakharov. Mechanisms proposed in the literature to produce inertia from vacuum effects (as in refs. [37] [38]) also presuppose inertia of the vacuum fields, and can thus not serve as primary mechanisms for inertia.

For the vacuum to be an agent for inertia it is necessary, in the first place, that a non-inertial observer be able to perceive enough details of its motion in the vacuum. The Lorentz invariance built into our theories leads to a vacuum that is, perforce, Lorentz invariant, so uniform motion cannot be detected through it. It is well known, however, that non-inertial motion raises from the vacuum a specter that can be sensed by the observer in different ways[39]. This phenomenon has so far been studied for only a limited class of simple motions. For example, for an observer on a collinear trajectory of constant-acceleration, a, (hyperbolic motion) this avatar of the vacuum is the Unruh radiation: a thermal bath the observer finds itself immersed in, of temperature $T = a/2\pi$ ($\hbar = 1$, $c = 1$, $k = 1$)[39][40]. Circular, highly relativistic motions have been discussed, e.g. in refs. [41]-[46] where it is found that a single parameter, $a = \gamma^2 v^2/r \approx \gamma^2/r$, still determines the spectrum of the incarnation of the vacuum (γ is the Lorentz factor); this is quasi-thermal with effective temperature $T = \eta a/2\pi$, where η is of order unity and depends somewhat on the frequency. For general motions, hardly anything is known about the radiation. It is clear that the effect must be a nonlocal functional of the whole trajectory, because the relevant wavelengths and frequencies of the radiation may be of the order of scale lengths and frequencies, respectively, that characterize the motion. (For stationary motions, such as the two described above, all points are equivalent, so the Unruh-like radiation appears to depend only on "local" properties. However, the non-local information on the stationarity of the trajectory enters strongly.)

While the Unruh-like radiation may well serve as a marker for non-inertial motions it is still difficult to implicate it directly in the generation of inertia: 1. It is not clear that it carries all the information on the motion needed to produce inertia. For example, even for hyperbolic motion, can the direction of its acceleration be told by the accelerated observer (it should be remembered that the radiation is characterized by more than just its spectrum. For example, a finite size observer can compare the radiation in its different parts.) 2. If inertia is local–as it is to a very good approximation in the non-MOND regime–it has to adjust instantaneously to the state of motion. The latter may change however on time scales that are short compared with the typical period of the Unruh-like radiation. In the MOND regime there is no experimental indication that inertia is local; on the contrary, as mentioned before, theoretical arguments point to nonlocal MOND inertia.

How does MOND fit into this, and, in particular, how can the connection with cosmology be made? When the acceleration of a constant-a observer becomes smaller than a_0, the typical frequency of its Unruh radiation becomes smaller than the expansion rate of the Universe, the Unruh wavelength becomes larger than the Hubble distance, etc. [30][3]. We expect then some break in the response of the vacuum when we cross the a_0 barrier. What is the Unruh radiation seen by a non-inertial observer in a nontrivial universe? We know that even inertial observers in a nontrivial universe find themselves immersed in radiation arising from the distortion of the vacuum. The simplest and best-studied case is that of a de Sitter universe in which all inertial observers see a thermal spectrum with a temperature $T_\Lambda = (\Lambda/3)^{1/3}/2\pi$ [47], where Λ is the cosmological constant characterizing the de Sitter cosmology. It was shown in refs. [48][49] that an observer on an hyperbolic trajectory, in a de Sitter universe, also sees thermal radiation, but with a temperature

$$T(a) = \frac{1}{2\pi}(a^2 + \Lambda/3)^{1/2}. \tag{5}$$

If inertia is what drives a non-inertial body back to (some nearby) inertial state, striving to annul the vacuum radiation–here, for hyperbolic motion, to drive T back to T_Λ–then $T - T_\Lambda$ is a relevant quantity. (With cosmology fixed, the best that inertia can do is drive T to T_Λ; in the cosmological context it also strives to drive T_Λ to zero.) We can write

$$2\pi(T - T_\Lambda) \equiv 2\pi\Delta T = a\hat{\mu}(a/\hat{a}_0), \tag{6}$$

with

$$\hat{\mu}(x) = [1 + (2x)^{-2}]^{1/2} - (2x)^{-1}, \tag{7}$$

and $\hat{a}_0 = 2(\Lambda/3)^{1/2}$. The quantity ΔT behaves in just the manner required from MOND inertia[1] $[\hat{\mu}(x \ll 1) \approx x, \hat{\mu}(x \gg 1) \approx 1-(2x)^{-1}]$ with $a_0 = \hat{a}_0$ naturally identified with a cosmological acceleration parameter. (This need not be the effective form of μ for trajectories other than hyperbolic; in mondified inertia there is no μ in the theory itself, and a different form of μ may apply, for instance, to circular orbits[3][25].) While this observation is interesting and suggestive, I cannot tell whether it is germane to MOND, because it is not backed by a concrete mechanism for inertia, and because I cannot generalize the observation to more general motions.

In de Sitter space-time the expansion rate, the space-time curvature, and the cosmological constant are one and the same. These parameters differ from each other in a general Friedmanian universe, and so the above lesson learnt for the de Sitter case does not tell us which of the cosmological acceleration parameters is to be identified with a_0 in the real universe.

Recall that, in MOND, inertia vanishes in the limit $a_0 \to \infty$. In the above picture this qualitative tenet of MOND is effected because the limit corresponds to $\Lambda \to \infty$, or $H_0 \to \infty$, etc.; so, the Gibbons-Hawking-like radiation due to cosmology swamps the thermal effects due to non-inertial motion: the difference between inertial and non-inertial observers is effaced in this limit.

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