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DYNAMICS OF ELLIPTICAL GALAXIES AND OTHER SPHEROIDAL COMPONENTS

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1. INTRODUCTION

Elliptical galaxies are the simplest type of galaxy and therefore the natural starting point for studies of galaxies in general. Furthermore, it has often been argued (e.g. de Vaucouleurs 1959, Ostriker 1977) that at the heart of every disk galaxy there sits a small elliptical, the bulge, around which the disk that now dominates the light distribution has been assembled. So it is possible that ellipticals and bulges are, so to speak, the founding fathers of the realm of the nebulae, and as such may have profoundly influenced the form of the components into which the majority of the luminous matter in the Universe subsequently settled.

For many years neither elliptical galaxies nor bulges received the attention warranted by their intrinsic interest. There were two reasons for this. The subsidiary reason was that it was widely felt that elliptical galaxies are so simple that their structures could be inferred by Plato's preferred method of research—pure thought. Surely these systems were isothermal bodies that rotate more or less rapidly according to their degree of equatorial-flattening. Galaxies of this type did not seem mysterious. However, the principal reason why ellipticals tended to be neglected in favor of disk galaxies was the great labor involved in obtaining reliable photometry or measuring their mean and

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random velocities. For practical purposes, and despite the pioneering work of de Vaucouleurs (1948) and Minkowski (1961), both of these types of measurements had to wait until the advent of computer-assisted plate-scanning machines, high-quantum efficiency spectrometers, and computers capable of extracting velocity information from the spectra.

Since 1976, a considerable body of photometric and kinematic data concerning elliptical galaxies has been assembled by astronomers around the world. And just as the merest blade of grass is discovered to be a superbly delicate structure as soon as it is examined under a microscope, so too have the simplest of galaxies proven to be remarkably complex. They are not isothermal and they do not rotate in proportion to their flattening. In this review, it is not possible to say what they are, because theory now lags behind observation in its efforts to coordinate the many observational data available into a self-consistent picture of elliptical galaxies. But the outlines of what may become the standard picture can be descried, and the observational facts can be stated.

The bulges of disk galaxies are only now coming under concerted attack by the observers, but the first results are fascinating. Much hangs on the question of whether bulges really are just small ellipticals, or whether they represent an independent type of beast. For if it can be shown that they form a subspecies of elliptical, this would constitute strong evidence that elliptical galaxies and bulges are the fundamental systems toward which theories of galaxy formation should be directed. But if observation shows that bulges are quite unlike ellipticals, this might indicate that ellipticals are, as Toomre (1977) has argued, formed by the merging of disk galaxies, which then become the basic products of galaxy formation in the primitive Universe.

The range of topics covered in this review has been severely restricted by the availability of space. It has not been possible to include a discussion of the formation and evolution of spheroidal components, or even to give a complete review of the literature concerning their equilibria. Additional information can be found in the reviews of Gott (1977, 1980), Freeman (1975, 1977) and Binney (1980a).

The article is organized as follows. Section 2 covers the basic stellar dynamical concepts. Section 3 is concerned with the radial structure of spheroidal components; Section 3.1 describes theoretical models against which the observations that are reviewed in Section 3.2 may be measured. Section 4 is concerned with the shapes of spheroidal components. Section 4.1 reviews the theoretical situation regarding axisymmetric and triaxial models, and Section 4.2 describes observations relevant to determining the shapes of spheroidal components and the dynamical processes that underlie their figures. Section 5 sums up and looks to the future.

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2. BASIC CONSIDERATIONS

Since before Baade (1944) resolved the brightest stars in the bulge of M31, it has been recognized that spheroidal components consist of swarms of stars that move about in the gravitational field that is generated by themselves and any other components that have an appreciable density in the same volume of space. Observations show that typical stellar speeds exceed 200 km s^{-1}, so that a star completes a half-orbit of semimajor axis r kpc in

$$T_{cr} = 10^7 r \text{ y} \approx 3 \times 10^7 \text{ y}. \tag{1}$$

The effective smoothness of the gravitational field in which the stars of a galaxy move may be measured by comparing the two-body relaxation time T_{2B} to T_{cr}. T_{2B} is the time typically required for a star in the system to be deflected from the path it would follow in the smooth potential by encounters with other stars. If the system is only modestly concentrated toward its center and is made up of N equal point masses, a simple application of the virial theorem and the expression for T_{2B} given by Spitzer & Härm (1958) shows that

$$T_{2B} \approx \frac{N}{30 \ln N} T_{cr}. \tag{2}$$

If the constituent masses are unequal, N in this formula should be set equal to the total mass of the system divided by the mass of the heaviest point masses. If we take $N > 10^{10}$, we may conclude that in the main body of a spheroidal component $T_{2B} > 10^8 \times T_{cr} > 10^{14}$ y, so that the trajectories of stars may be computed using a smooth mean field for times longer than the Hubble time T_H. However, one should note that there are two circumstances in which two-body effects can play a role in the dynamics of spheroidal components. The first is when the effective value of N is smaller than 10^5. For example, a massive globular cluster may contain $10^7\ M_\odot$, i.e. 10^{-3} of the mass of a typical spheroidal component, with the result that a system of globular clusters orbiting through a spheroidal component corresponds to $N \lesssim 10^3$ and may suffer appreciable evolution within a Hubble time (Tremaine et al. 1975). The second case is when a spheroidal component is so centrally concentrated that its nucleus may become dynamically separate from the main body of the galaxy. The crossing time T_{cr} may be very small for stars in the nucleus (e.g. $T_{cr} < 10^5$ y in the nucleus of M31) and N may be less than or of order 10^7, so that $T_{2B} < T_H$ there. Spitzer & Saslaw (1966) have discussed the significance of this situation for the formation of active galactic nuclei.

If one confines oneself to consideration of the main body of the galaxy, the unimportance of two-body evolution enables one to describe the dynamics of the system with the single-particle distribution function $f(\mathbf{x}, \mathbf{v})$ that gives the

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mass density of stars at the point ($\mathbf{x}$, $\mathbf{v}$) in phase space. If f is a differentiable function of $\mathbf{x}$ and $\mathbf{v}$, then it evolves in time according to the collisionless Boltzmann or Vlasov equation:

$$\mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = -\frac{\partial f}{\partial t}. \tag{3}$$

One is usually interested in the steady states of galaxies, when $\partial f / \partial t$ might be supposed to vanish, at least when f is referred to some suitable rotating coordinate system. Unfortunately, a distribution function f that is initially smooth may become with the passage of time less and less smooth in certain parts of the space, with the result that for many potentials differentiable time-independent solutions of Equation (3) do not exist. When the f that satisfies Equation (3) becomes rough in this way, one must distinguish between the fine-grained distribution function f and the coarse-grained distribution function f_c, obtained by averaging f over macroscopic regions in phase space. When f does not become rough, and the right-hand side of Equation (3) vanishes for some differentiable f, Equation (3) then states that f is constant along the curves in phase space that are followed by the representative points of stars as the stars orbit in the potential. That is, the time-independent Vlasov equation states that f is an integral of the equations of stellar motion. If I_1 ($\mathbf{x}$, $\mathbf{v}$) , . . . , $I_n(\mathbf{x}, \mathbf{v})$ is a complete set of integrals, then

$$f = f(I_1, \ldots, I_n), \tag{4}$$

which is known as Jeans' theorem. Conversely, any integral or function of any integrals that happen to be known gives solutions to the time-independent Vlasov equation.

Jeans' theorem raises the thorny question of how many integrals there are in a complete set. It is important to distinguish between isolating integrals and nonisolating integrals. An isolating integral is one for which the equation

$$I(\mathbf{x}, \mathbf{v}) = C, \tag{5}$$

where C is a constant, defines a smooth five-dimensional hypersurface in phase space. It is clear that the minimum number of isolating integrals is one, because the energy $E(\mathbf{x}, \mathbf{v})$ is always such an integral, and the maximum number is five, because the intersection of the N five-dimensional hypersurfaces associated with each isolating integral must contain the one-dimensional space of an individual orbit. However, very few potentials admit as many as five isolating integrals. The usual number of isolating integrals is three, one for each pair of canonical coordinates.

When there are three or more isolating integrals, the orbit of a star is quasi-periodic; that is, the evolution of any phase space coordinate may be expressed as a Fourier series

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$$x(t) = \sum_{n,m,\ell=-\infty}^{\infty} A_{nm\ell} \exp[i(n\omega_1 + m\omega_2 + \ell\omega_3)t], \tag{6}$$

where ω_1, ω_2, and ω_3 are frequencies characteristic of the orbit and nearly all the power is concentrated in the terms involving small n, m, and ℓ (Binney & Spergel 1982). Four or five isolating integrals arise when two or more of the frequencies ω_i are commensurable. An example of this phenomenon is given by motion in the Kepler potential $\Phi \sim 1/r$. Then the orbital frequency is the only independent frequency, and there are five isolating integrals.

In a general potential, some (often the great majority) of the orbits are quasi-periodic and admit three isolating integrals. These are the regular orbits. But there are usually some orbits, called irregular or stochastic orbits, that are not quasi-periodic. These orbits cannot admit three isolating integrals because their representative points in phase space occupy a volume that has more than three dimensions, and therefore cannot be described as the intersection of three five-dimensional hypersurfaces. However, these orbits do not pass close to every point in the five-dimensional hypersurface of their energy integral, i.e. they are not ergodic in the sense of classical statistical mechanics. The so-called KAM theorem (e.g. Moser 1977) assures us that there are always regions of the energy hypersurface that are forbidden to them because that space is occupied by regular orbits. And one finds that even in parts of the phase space where irregular orbits are in the majority, a given irregular orbit will spend the majority of its time in certain areas (Ichimura & Saito 1978, Goodman & Schwarzschild 1981, Binney 1982b). This complex orbital structure prevents the fine-grained distribution function f, for which the Vlasov equation (3) holds, from reaching a steady state.

It is uncertain what role the irregular orbits play in the structure and evolution of spheroidal components. In particular, we do not know what features of the potential Φ determine what proportion of all orbits are irregular. But in any galaxy in which an appreciable fraction of the stars are on irregular orbits, the coarse-grained distribution function $f_c(\mathbf{x}, \mathbf{v})$, which is the quantity of astronomical interest, will not obey the time-independent Vlasov equation and hence will not be a function of $(\mathbf{x}, \mathbf{v})$ only through the integrals of stellar motion. Therefore Jeans' theorem should be used more cautiously than perhaps it has been in the past. Furthermore, irregular orbits may evolve in a systematic way on time scales that are long compared to the dynamical time of the system, but not longer than the Hubble time. If this is so, they may drive galactic evolution in a way that has yet to be fully explored (Sanders, in preparation).

The ideal basis for the interpretation of observations would be an array of stellar dynamical models deriving from exact solutions of the Vlasov equation (3). Unfortunately we are far from possessing such a hoard of treasure and we have to confine ourselves, for the most part, to statements about the properties

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such models would have if we were able to construct them. Constraints of this type are obtained by integrating multiples of the Vlasov equation over all velocities (or over both velocities and space) to obtain moment equations. Thus when one multiplies Equation (3) by 1 or $\mathbf{v}$ and integrates over all velocities, one obtains the equations that have been called the equations of stellar hydrodynamics by reason of their similarity to the equations of fluid flow. [A more convenient and less misleading designation might be the "Jeans equations" in honor of Jeans' (1922) investigation of them.] For future reference note that the equation obtained on multiplication of (3) by $\mathbf{v}$ is for a steady-state system of spherical symmetry

$$\frac{d \ln \rho\sigma_{\mathrm{r}}^2}{d \ln r} + 2\beta = -\frac{GM(r)}{r\sigma_{\mathrm{r}}^2} \equiv \frac{-v_{\mathrm{c}}^2(r)}{\sigma_{\mathrm{r}}^2} \tag{7}$$

where

$$\rho \equiv \int f \, d^3\mathbf{v} \tag{8a}$$

$$\sigma_{\mathrm{r}}^2 \equiv \langle v_{\mathrm{r}}^2 \rangle \equiv (1/\rho)\int f v_{\mathrm{r}}^2 \, d^3\mathbf{v} \tag{8b}$$

$$\beta \equiv 1 - \langle v_\theta^2 \rangle / \langle v_{\mathrm{r}}^2 \rangle \tag{8c}$$

$$M(r) = 4\pi \int_0^r \rho \, r^2 dr \tag{8d}$$

When the Vlasov equation (3) is multiplied by $x_i v_j$ and integrated over all velocities and spatial coordinates, the equations of the tensor virial theorem are obtained (Chandrasekhar 1964, Binney 1978a). This theorem is valuable because it states that the ratios $\langle \rho v_z^2 \rangle / \langle \rho v_x^2 \rangle$, etc., of the kinetic energies associated with the components of motion parallel to the three body-axes of the system depend on the shape of the system and the speed with which the figure rotates with respect to inertial space, but not on its radial density profile. In particular, if one knows the figure of a flattened axisymmetric galaxy, one immediately knows how much more kinetic energy is associated with motion parallel to the equatorial plane than perpendicular to this plane. Some of this additional kinetic energy will be associated with rotation, and the rest with anisotropy of the velocity dispersion tensor.

The tensor virial theorem has been used in recent years as the standard framework within which to analyze observations of the kinematics of spheroidal components. However, it should be realized that the popularity of the virial theorem arises not so much from its own merits, but because we lack realistic models of flattened spheroidal components. Consequently, we are obliged to reduce the wealth of information contained in the best recent observations of spheroidal components to the pair of numbers that can be accommodated by the tensor virial theorem. Section 4 discusses the origin of this unfortunate situation.

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3. RADIAL STRUCTURE OF SPHEROIDAL COMPONENTS

The variation of mass density, luminosity density, and velocity dispersion as a function of radius are most easily discussed in terms of spherical models. These are much easier to construct than nonspherical models, and neither observation nor theory indicate that the radial profiles of nonspherical galaxies are affected in important ways by the shapes of the systems. In particular, the works of Saaf (1968) and Richstone (1981) indicate that the total angular momentum of a star is approximately conserved when it orbits in a mildly nonspherical potential.

3.1 *Model Galaxies*

If a galaxy is spherical, stars orbiting in its potential are constrained by four independent isolating integrals—the three components of the angular momentum vector **J** and the energy—and we may invoke Jeans' theorem to construct models by taking f to be an arbitrary positive function of these four integrals. If the galaxy is not only spherical, but also spherically symmetric in all its properties, f can depend on J_x, J_y and J_z only through the combination $J^2 = J_x^2 + J_y^2 + J_z^2$ so f is then of the form $f = f(E,J)$.

SYSTEMS HAVING $f(E)$ For many years elliptical galaxies have been discussed in terms of models whose distribution functions depend only on E. These models are interesting, but it is important to recognize that they constitute a narrowly restricted class of possible spherically symmetric galaxies, and it is unlikely that Nature confines herself to models of this type. The majority of these models are modifications of the isothermal sphere, whose distribution function is simply

$$F_{\mathrm{I}}(E) = (2\pi\sigma^2)^{-3/2}\rho(0)\exp\{[\Phi(0) - E]/\sigma^2\}, \tag{9}$$

where $\Phi(0)$ is the potential energy at the center of the system. Integrating f_{I} over all velocities yields the density at radius r as

$$\rho(r) = \rho(0)\exp\{[\Phi(0) - \Phi(r)]/\sigma^2\}. \tag{10}$$

If the system is self-gravitating, one obtains on solving Poisson's equation with ρ replaced by (10),

$$\Phi(0) - \Phi(r) \approx 2\sigma^2 \ln(r/r_c) \qquad (r >> r_c), \tag{11}$$

where

$$r_c = 3\sigma[4\pi G\rho(0)]^{-1/2} \tag{12}$$

is the "core radius" at which the projected density falls to very nearly ½ of its value at the center. Substituting Equation (11) into (10), one sees that at large

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radii, $\rho \sim r^{-2}$. If the system is not self-gravitating, but sits in the potential of another isothermal population whose velocity dispersion σ_h differs from that of the first system, one has from equations (10) and (11) that at large radii the density of the first population falls off as $\rho \sim r^{-2(\sigma_h/\sigma)^2}$. Gunn (1977) has argued that the brightness profiles of galaxies may fall off more steeply than as r^{-2} because the velocity dispersion σ of the luminous stars is less than that (σ_h) of the mass-bearing halo population by a factor of order $\sigma/\sigma_h = 0.82$.

An alternative strategy for obtaining from $f_{\rm I}$ a system that looks like an elliptical galaxy is to truncate $f_{\rm I}$ at some maximum energy $E_{\rm t}$. Woolley (1954) simply set f equal to $f_{\rm I}$ for E less than $E_{\rm t}$ and to zero otherwise. King (1966), in parallel with Michie (1963), eliminated the discontinuity in Woolley's distribution function at $E_{\rm t}$ by defining for $E < E_{\rm t}$.

$$f_{\rm K}(E) = f_{\rm I}(E) - f_{\rm I}(E_{\rm t}) = (2\pi\sigma^2)^{-3/2}\rho_1\{\exp[(E_{\rm t} - E)/\sigma^2] - 1\} \qquad (13)$$

Wilson (1975) subsequently eliminated the discontinuity in the gradient of $f_{\rm K}$ at $E_{\rm t}$ by defining for $E < E_{\rm t}$,

$$f_{\rm W} = (2\pi\sigma^2)^{-3/2}\rho_1\{\exp[(E_{\rm t} - E)/\sigma^2] - 1 - (E_{\rm t} - E)/\sigma^2\}. \qquad (14)$$

Proceeding in this way one may generate a sequence of models, all of which are effectively isothermal near their centers, where $E << E_{\rm t}$, but which have nonisothermal envelopes.

Hunter (1977) has shown that the structure of the envelopes of these models depends sensitively and in a paradoxical way on the detailed form of the distribution function near the tidal cutoff. In particular, Wilson spheres, which have more heavily truncated distribution functions than King models, have much more extensive envelopes. It follows from this state of affairs that one cannot say a priori whether tidal encounters between galaxies lead to tidal truncation of the galaxies or to distension of their envelopes. The observations discussed below suggest the latter (Kormendy 1977; but see Strom & Strom 1978d).

As is discussed in Section 3.2, the brightness profiles at the centers of elliptical galaxies tend to be more peaky than the projected density of an isothermal sphere. Therefore it is interesting to study model galaxies whose projected density profiles have a cusp at the center. Eddington (1916) showed how to find the $f(E)$ that generates a galaxy of any given radial density profile, and one may apply this apparatus (Binney 1982a) to find the distribution function $f_{1/4}(E)$ that generates the galaxy whose projected surface density obeys de Vaucouleurs' (1948) $r^{1/4}$ law of surface brightness. One finds that $f_{1/4}(E)$ rises steeply at energies that correspond to stars confined to the center of the galaxy. It is this abundance of tightly bound stars that gives rise to the central density peak and velocity dispersion depression (Bailey & MacDonald 1981) that are characteristic of the $r^{1/4}$ model. Binney (1982a) has proposed a theoretical interpretation of this model.

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SYSTEMS HAVING $f(E,J)$ Eddington (1914; see also Shiveshwarkar 1936) considered simple models, based on $f = f(E, J)$, that can probably not be used to describe any real system, but which do illustrate the way in which velocity dispersion anisotropy affects the structure of spherical systems. The distribution functions of these Eddington models are of the form

$$f_E(E,J) = f_I(E) \exp[-J^2/(2r_a^2\sigma^2)]. \tag{15}$$

The part of Eddington's distribution function (15) that depends on J causes the density at radius r to drop from the value [Equation (10)] associated with the isothermal sphere to

$$\rho(r) = \frac{\rho(0)}{1 + (r/r_a)^2} \exp\{[\Phi(0) - \Phi(r)]/\sigma^2\}, \tag{16}$$

and causes each tangential component of squared velocity dispersion to diminish by a fraction β of the radial component of $\boldsymbol{\sigma}$, where

$$\beta = 1 - \sigma_\theta^2/\sigma_r^2 = [(r_a/r)^2 + 1]^{-1}. \tag{17}$$

Thus the velocity dispersion tensor in an Eddington model is isotropic at the center and wholly anisotropic at large radii. The radial component of velocity dispersion equals the constant σ at all radii.

The outermost part of most Eddington models is an envelope in which $\rho \sim r^{-2}$. This envelope bears a superficial resemblence to the outermost portion of the isothermal sphere, but it is actually of an entirely different nature because the circular velocity $v_c(r)$ in an Eddington model tends to zero as r increases, rather than to a finite constant as in the isothermal sphere ($v_c = \sqrt{2}\sigma$). On the other hand, the radial component of velocity dispersion in an Eddington model is always equal to σ. Therefore the gravitational attraction of stars interior to r slows or deflects the motion of a star with typical speed σ less and less as r increases, and stars far from the core of an Eddington model execute giant oscillations in radius. The system becomes, in fact, a kind of stellar traffic jam in which each star moves with more or less uniform velocity on a radial path. The $\rho \sim r^{-2}$ increase of density toward the center has less to do with gravity and dynamics than with congestion of these trajectories. In terms of Equation (7), one may say that at the outside of an Eddington model the term on the right-hand side of this equation has dropped out, leaving the structure to be determined by a balance between the two terms on the left.

Color gradients (de Vaucouleurs 1961, Strom & Strom 1978a,b,c) in principle offer a way of constraining the degree of radial velocity anisotropy in a galaxy. If all the stars of a galaxy were on circular orbits, so that the galaxy could be considered to be constructed of infinitesimally thin spherical shells, color changes could be perfectly sharp in three-dimensional space and moderately sharp when projected onto the sky. If, on the other hand, the system

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resembled the asymptotic portion of an Eddington model; all stars would contribute equally to the light at each radius, and no color gradient would be possible.

Michie (1963) studied the models that are related to King models in the same way that Eddington models are related to the isothermal sphere. The distribution function of a Michie model is

$$f_M(E,J) = f_K(E)\exp[-J^2/(2r_a^2\sigma^2)]. \tag{18}$$

For sufficiently large anisotropy radius r_a, a Michie model behaves like a King model in that it has a "tidal radius" at which the density goes to zero. When r_a is small, the Michie model has an infinite envelope that resembles the envelope of an Eddington model. The Michie models that are of the greatest interest from the point of view of modeling real galaxies are those that have finite tidal radii. In these models the anisotropy parameter β never comes close to unity.

3.2 *Observations of Radial Structure*

PHOTOMETRY Unfortunately, many spectroscopic data cannot yet be interpreted dynamically because the requisite photometry is lacking. Furthermore, the interpretation of the available photometry is more controversial than is the interpretation of the spectroscopic observations.

It is convenient to divide the brightness profile of a typical elliptical galaxy into an inner part that extends out to $R_I = 5\sigma_s$, where $\sigma_s \sim 0.75''$ is the dispersion of the central Gaussian component of a typical seeing disk, an intermediate part that runs from this radius out to the de Vaucouleurs radius R_D (the radius where the B brightness falls to $\mu = 25$ mag/arcsec^2), and the part that lies outside R_D. The true brightness distributions of spheroidal systems in their inner and outer parts are very difficult to measure and are correspondingly uncertain. The profiles in the intermediate region are, by contrast, fairly well determined.

The radial brightness profiles of a sample of 17 elliptical galaxies studied by King (1978) are very similar to one another in the intermediate region, though the profiles do show some genuine individuality (see Figure 1 of Kormendy 1977). The ellipticity of King's galaxies, which varies from $\epsilon = 0$ to $\epsilon = 0.4$, appears not to affect the mean radial brightness profile.

A variety of fitting functions provide satisfactory fits to the profiles of King's galaxies in the intermediate range of radii, although Kormendy (1977) concludes that the $r^{1/4}$ law provides a more convenient overall fit than the Hubble-Reynolds or King profiles. A particularly striking example of the quality of fit to observations of elliptical galaxies that can be obtained with the $r^{1/4}$ law is provided by the extensive study by de Vaucouleurs & Capaccioli (1979) of the surface brightness of the E1 galaxy NGC 3379. They show that

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the $r^{1/4}$ law of surface brightness fits the observations of this galaxy to within 0.1 mag over a range of 9 mag in surface brightness.

The brightness profiles of galaxies below about 26 mag/arcsec^2 are rather problematical. The B-band sky brightness in these outer regions typically exceeds the galaxian surface brightness by a factor of more than 100, so that a small error in the choice of sky brightness level to be subtracted from the raw observations can lead to a large error in the derived galaxian brightness profile. Published photometry indicates that the behavior of brightness profiles beyond R_D is highly variable. Kormendy (1977) finds that the surface brightnesses of King's ellipticals always exceed that of the best-fitting $r^{1/4}$ law at large radii, and they sometimes exceed that of the best-fitting Hubble profile. The latter phenomenon occurs most commonly amongst galaxies that have companions, as if elliptical galaxies are distended rather than truncated by tidal encounters.

In a log-log plot, the brightness profiles of all of King's galaxies steepen fairly steadily with increasing radius. By contrast, certain supergiant galaxies studied by Oemler (1976), Dressler (1978), Carter (1978), Hoessel et al. (1980) and others have brightness profiles whose slopes in a log-log plot flatten at large radii. The designation cD, which is sometimes used rather loosely, is best confined to galaxies of this class.

The bulges of disk galaxies show a general similarity to elliptical galaxies (e.g. Kormendy 1977, Tsikoudi 1980) although there are differences in detail. Unambiguous information about bulges can only be obtained from the minor-axis profiles of edge-on galaxies, for only profiles of this type are uncontaminated by an uncertain contribution from the disk. The bulge of the edge-on Sb galaxy NGC 4565, which has been studied by Hegyi & Gerber (1977), Spinrad et al. (1978), and Kormendy & Bruzual (1978) cannot be fitted over the range $21 < \mu_v < 28$ by either a single Hubble law or a simple $r^{1/4}$ law. It is not known whether this phenomenon is widespread among the bulges of disk galaxies, although Burstein (1979) finds that the profiles of edge-on S0 galaxies also cannot be exactly fitted by the $r^{1/4}$ law.

It has been suggested (Freeman; report to NATO ASI, Cambridge 1980) that these differences may be due to distortion of the bulges by the gravitational field of the disk. It is also possible that all low-luminosity spheroidal components, including dwarf ellipticals, have profiles that differ systematically from those of giant ellipticals (Strom & Strom 1979).

The true shapes of the brightness profiles of spheroidal systems at radii comparable to the core of the point-spread function (PSF) imposed by seeing are highly controversial. Schweizer (1979, 1981) has recently studied the results of convolving ideal galaxy profiles with various model PSFs in some detail. His conclusions are as follows: (*a*) A seeing-convolved $r^{1/4}$ profile looks much like an unconvolved King profile. The apparent core radius of this profile is typically 3–4 times the dispersion σ_s of the Gaussian core of the PSF.

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(*b*) A seeing-convolved King profile looks like an unconvolved King profile of larger core radius. The ratio $r_{c,app}/r_c$ of the apparent to the true core radius does not fall to 1.25 until $r_{c,app} > 3.5\sigma_s$. (*c*) The predicted profiles are changed materially by the inclusion in the PSF of exponential wings or additional Gaussian components, such as those advocated by Brown (1974) and by de Vaucouleurs & Nieto (1979).

When one reviews the apparent core radii of the galaxies studied by King in the light of these results, one finds that the photometric data are in most cases unable to distinguish between the possibility that these galaxies have regions of constant density at their centers similar to that of a King model, or have volume densities of stars that rise towards a singularity of the type required to generate the $r^{1/4}$ law in projection. However, external evidence suggests that these galaxies are likely to have rather singular central densities: Schweizer (1979) notes that the spheroidal components of the Local Group galaxies M31 and M32 have central surface brightnesses that are 2–3 mag brighter than those inferred by fitting King models to King's sample of giant ellipticals. They are, however, very much in the range of central surface brightnesses inferred from King's sample by fitting $r^{1/4}$ profiles. This suggests that giant elliptical galaxies appear less centrally concentrated than M31 and M32 only because they are more distant and therefore less well resolved. Furthermore, neither counts of RR Lyrae stars and globular clusters toward the center of our Galaxy (Oort 1976) nor studies of the gas at the Galactic center (Lacy et al. 1979) provide any evidence that our Galaxy has a quasi-isothermal core.

Two spheroidal components whose central regions have been carefully studied are M87 and the bulge-nucleus of M31. Young et al. (1978b) obtained high signal-to-noise V-band observations of the central 80″ of M87. They found that the brightness distribution near the center of M87 cannot be fitted with a King model or a King model plus a point light source. De Vaucouleurs & Nieto (1979) have confirmed the photometry of Young et al. and concluded that M87 has less light in the radius range $r < 8''$ than the $r^{1/4}$ law fitted to the observations beyond $r = 8''$ would require. The ground-based observations of M31 by Johnson (1961), together with data collected by the balloon-borne telescope *Stratoscope II*, show (Light et al. 1974) that interior to $r = 20''$ the brightness profile of M31 is qualitatively similar to that of M87. Again there is a clearly defined shoulder in the brightness distribution somewhat outside the region where seeing markedly degrades ground-based observations. Interior to this shoulder the surface brightness first flattens off and then rises again steeply to a peak surface brightness that is determined by the PSF of the observations. The brightness profiles of M87 and M31 are associated with the velocity dispersion anomalies discussed below.

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SPECTROSCOPY Great strides have been made over the last six years in the absorption-line spectroscopy of early-type systems. Until a few years ago, very few systems had been observed spectroscopically even near their centers, where they are brightest. Furthermore, the few measurements that were available showed a distressingly wide spread of values for the same system when measured by different observers. The introduction of automatic algorithms for the reduction of spectra has transformed this situation.

Two quite independent methods of obtaining kinematic information from absorption-line spectra are now in use. The majority of workers use some variant of the Fourier quotient technique that was originally developed by Illingworth (1976) and Schechter (Sargent et al. 1977). Data collected in the extensive Harvard survey of galactic velocities in the Local supercluster (Tonry & Davis 1981a,b) have been analyzed with a cross-correlation algorithm (Tonry & Davis 1979). Efstathiou et al. (1980) have found that these two methods yield very similar results when applied to the same data. Terlevich et al. (1981) find that velocity dispersions obtained for one galaxy by different observers using different equipment and reduction techniques now agree to within the ~10% cited errors. Differential velocities within galaxies are commonly measured to an accuracy of 20 km s^{-1} or better.

Two types of data must be considered. The ideal study yields the velocity dispersion and mean velocity as a function of position in the galaxy from the center far out into the halo. Studies that approach this ideal have now been carried out on a few dozen galaxies of type E and S0 (Sargent et al. 1977, 1978, Young et al. 1978a, Schechter & Gunn 1979, Efstathiou et al. 1980, Davies 1981, Carter et al. 1981, Illingworth & Schechter 1981, Kormendy & Illingworth 1982, Kormendy 1981a,b, Fried & Illingworth, in preparation, Davies et al., in preparation). A much less time-consuming observation involves measuring the systematic velocity and the velocity dispersion from a single spectrum of the light from the center of the galaxy. Measurements of this type have now been obtained for a few hundred galaxies of type E and S0 (Faber & Jackson 1976, Schechter 1980, Tonry & Davis 1981a,b, Faber et al., in preparation).

These data show that the central line-of-sight velocity dispersion σ_{v0} is tightly correlated with total luminosity L. Faber & Jackson (1976) found the velocity dispersion of a sample of 24 E and S0 galaxies to be well represented by the law $L \sim \sigma_{v0}^4$. Several subsequent investigations (Sargent et al. 1977, Schechter & Gunn 1979, Schechter 1980, Terlevich et al. 1981, Tonry & Davis 1981b) have confirmed that σ_{v0} and L are well correlated, although the value of the slope n when the correlation is fitted to the power law $L \sim \sigma_{v0}^n$ has varied in the range $3 < n < 5$. Probably the correlation cannot be adequately fitted by a single power law over the full range of absolute magnitudes

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($-23 < M < -15.5$) for which corresponding velocity dispersions are now available. Tonry (1981) finds that at faint luminosities, σ_{v0} rises more steeply than $L^{1/4}$, and Efstathiou et al. (1980) and Malmuth & Kirshner (1981) find that luminous cD galaxies have smaller velocity dispersions than the extrapolation of $\sigma_{v0} \sim L^{1/4}$ would suggest.

The more luminous an elliptical galaxy is, the more strongly lined is the light it emits (e.g. Faber 1973). A useful measure of line strength is the Mg_2 index defined by Faber et al. (1977). Terlevich et al. (1981) have investigated the distribution of the representative points of galaxies in the three-dimensional space defined by L, σ_{v0} and Mg_2. They argue that in this space all but 4 of a sample of 24 galaxies lie in a long flat volume like that occupied by a ruler; that is, Terlevich et al. conclude that elliptical galaxies form a two-parameter family. Tonry & Davis (1981b) have discussed the distribution of a sample of more than 50 ellipticals in a similar three-dimensional space. They conclude that the galaxies are distributed in this space within a long cylinder that has three full dimensions, and they go on to argue that the analysis of Terlevich et al. suggested that part of that sample occupies a two-dimensional space only because the analysis failed to eliminate the elongation of the cylinder occupied by the sample. Terlevich et al. reply that the errors in the data of Tonry & Davis are too large for two-dimensionality to be detectable. Work is now in progress on observations that should resolve this controversy. If the main finding of Terlevich et al. is confirmed, it will be interesting to see whether the second parameter among elliptical galaxies is, as Terlevich et al. suggest, true ellipticity.

Whitmore et al. (1979) have measured the central velocity dispersions of 21 spiral galaxies and plotted their results against estimates of the absolute bulge magnitudes of these systems, which span the range $-17.5 > M_B > -22.7$. Their results are consistent with $L \sim \sigma_{v0}^4$ over this range, although the dispersion at a given L may be 15% lower than in an equivalent elliptical. However, the bulges of spirals cannot be self-gravitating at all radii because observations of neutral hydrogen in the disks surrounding them show that the circular velocity $v_c \approx \sqrt{3}\sigma_{v0}$ at large radii (Whitmore et al. 1979). If the bulges were everywhere self-gravitating, one would have $v_c < \sqrt{2}\sigma_{v0}$ far from the center. Nonetheless the bulges probably are self-gravitating at their centers.

It is remarkable that σ_{v0} should be tightly correlated with L, for σ_{v0} depends on the structure of the galaxy in a small region that contributes very little of the total light. Furthermore, the velocity dispersion in most elliptical galaxies declines from the nucleus outward (see Figure 3 of Illingworth 1981), so that the velocity dispersion that is so well correlated with luminosity is not the velocity dispersion of the stars that contribute most of the light. Possible explanations of how σ and L can be correlated have been offered by Sargent

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et al. (1977) and by Tonry (1981), but before assessing the plausibility of these pictures it is necessary to clarify one's ideas concerning the relationship between a galaxy's brightness and velocity dispersion profiles and its mass-to-light ratio.

Suppose the velocity dispersion tensor in a spherical galaxy of known central velocity dispersion $\boldsymbol{\sigma}$ is isotropic. Then two simple methods will lead from surface photometry of the galaxy to a fairly reliable estimate of the mass-to-light ratio near the center of the galaxy.

1. Fit a King model to the central brightness distribution to determine the apparent core radius r_c and central surface luminosity I_0 of the fit. Then substitute these into the formula (King & Minkowski 1972)

$$M/L = 9\sigma_{v0}^2/(2\pi G I_0 r_c). \tag{19}$$

The effects of seeing on results obtained with this formula are unimportant, because seeing transforms the brightness profile of a King model into a profile that closely resembles that of a King model of larger core radius but equal velocity dispersion (Schweizer 1979).

2. Fit an $r^{1/4}$ profile to the photometry to determine the effective radius r_e and the associated surface luminosity I_e at r_e, and then substitute them into the formula

$$M/L = 0.201\sigma_{v0}^2/(G I_e r_e). \tag{20}$$

This formula is derived by assuming that the observed central line-of-sight velocity dispersion is equal to the peak line-of-sight velocity dispersion 0.469 GM/r_e predicted for an $r^{1/4}$ model of total mass M by Bailey & MacDonald (1981), and by using the standard relation $L = 7.22 I_e \pi r_e^2$ for the total light of the $r^{1/4}$ profile. Equation (20) is to be preferred to the similar relationship that relates M/L to the luminosity-weighted velocity dispersion $\bar{\sigma}$ of an $r^{1/4}$ model (Poveda et al. 1960), because the latter is not what is usually measured. (Half the light of the $r^{1/4}$ model is received at surface brightnesses fainter than $(I_e/4\pi) \sim 23$ mag/arcsec^2.) When, as sometimes happens, σ_{v0} is used in Poveda's formula in place of $\bar{\sigma}$, the mass-to-light ratio recovered is too great by a factor of 2 (Michard 1980).

Schechter (1980) has used method 1 and Michard (1980) has used method 2 to derive M/L for numbers of early-type galaxies. They find $5 < (M/L_B) < 12$ in solar units. The indications regarding the dependence of M/L on L are confusing. On the one hand, two arguments suggest that M/L should increase with L: Tinsley (1978) showed that M/L should increase as $L^{0.13}$ because the stars of luminous galaxies are metal rich; and if L increases with r_e less rapidly than r_e^2 (Michard 1979, Kormendy 1977), L can rise as σ^4 only if M/L rises with L. On the other hand, neither Schechter nor Michard (1980) were able to find any clear indications in the data that M/L increases

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with L, notwithstanding the earlier work of Faber & Jackson (1976), who found $M/L \sim L^{0.5}$.

Simien et al. (1979) and Whitmore et al. (1979) find similar values of M/L_B for the bulges of spiral galaxies.

For many galaxies, velocity dispersions are available to large distances from the center, and more sophisticated interpretation of the data is called for than is possible by merely fitting these systems with King or $r^{1/4}$ models. The most general model of a spherically symmetric galaxy involves four functions of radius: the luminosity density $\ell(r)$, the radial component of the velocity dispersion tensor $\sigma_r(r)$, the anisotropy parameter $\beta(r)$ defined by Equation (8c), and the mass-to-light ratio $M/L = A(r)$. The luminosity density may be determined from good photometry, and Equation (7) relates one of the remaining unknown functions to the other two. Hence if one unknown function is chosen a priori, for example by setting $\beta =$ constant or A = constant, the other may, in principle, be determined from the observed run of velocity dispersion with radius.

Sargent et al. (1978) chose to interpret their combined spectroscopic and photometric observations of the inner 80″ of M87 in terms of a model in which $\beta = 0$ and A is allowed to vary. They concluded that under these hypotheses the mass-to-light ratio A has to increase markedly towards the center, where $M/L \sim 60$ is about a factor 10 higher than in the body of the galaxy. They suggest that this may be due to the presence of a black hole at the center of the galaxy.

One may argue, however, that if either A or β has to be arbitrarily set equal to a constant, it makes more sense to fix A and to allow β to vary, rather than vice versa. All types of elliptical galaxies seem to have similar overall mass-to-light ratios despite the large ranges of metallicity and stellar density among these systems. This suggests that there is a standard mix of stars out of which elliptical galaxies are made. And the studies of rotation in giant elliptical galaxies (see below) show unambiguously that the velocity dispersion tensors in flattened galaxies are not isotropic, and their anisotropy has nothing to do with rotation. Therefore we must examine carefully the possibility that the velocity dispersion tensors in all elliptical galaxies, including spherical systems, are anisotropic.

Duncan & Wheeler (1980) have shown that the Sargent et al. data for M87 are fitted moderately well by a point light source plus an Eddington model with a central mass-to-light ratio $M/L_v = 7.1$. However, the model proposed by Duncan & Wheeler has $\beta > 0.9$ for $r > 1.7$ kpc and $\beta > 0.99$ for $r > 5.5$ kpc. Such highly anisotropic distributions are not very plausible.

This situation prompted Binney & Mamon (1982) to investigate the problem posed by observations of $\sigma_v(r)$ and $I(r)$ more generally. They show that if one presumes that the mass-to-light ratio A is (an initially unknown) constant, a

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given run of $\sigma_v(r)$ and $I(r)$ determine A as an eigenvalue associated with the system of equations from which one obtains $\beta(r)$. If $\beta(r)$ is physically plausible (in particular if $\beta < 1$ at all r), one has then recovered the unique spherical model that has constant mass-to-light ratio and is compatible with the observations. Binney & Mamon apply this technique to the Sargent et al. observations of M87. They recover a model in which $M/L_v = 7.6$, $\beta < 0.9$ everywhere, and $\beta \sim 0.4$ in the body of the galaxy.

The central region of M31 poses a problem that is in many respects similar to that posed by M87. Setting $\beta = 0$ and assuming constant mass-to-light ratio, Ruiz (1976) concluded from the photometry of Johnson (1961) and Light et al. (1974) that the velocity dispersion σ_n at the center of M31 should be smaller than that (σ_b) at 10″ by a factor of about 0.6. However, Morton et al. (1977) and Whitmore (1980) find that σ_n is if anything larger than σ_b. As Tremaine & Ostriker (1982) point out, these results can be understood if (*a*) $\beta = 0$ and the mass-to-light ratio in the nucleus is greater than in the bulge by a factor of about 5, or (*b*) if $\beta \sim 1$ and the mass-to-light ratio in the nucleus is smaller than in the bulge.

The mass-to-light ratios in spiral galaxies are known to increase from the centers outward (e.g. Rubin et al. 1978) and one might anticipate a similar increase of M/L toward the outer regions of elliptical galaxies. Unfortunately the same uncertainty as to the behavior of the anisotropy parameter β that makes interpretation of observations of M87 difficult bedevils attempts to demonstrate observationally that elliptical galaxies have large M/L far from their centers. The most convincing evidence that at least some ellipticals do show this effect is provided by Dressler's (1979) observations of the cD galaxy in the cluster Abell 2029 and by observations of IC 2082 by Carter et al. (1981). Dressler found that the velocity dispersion in his cD galaxy rises from 380 km s^{-1} at the center to about 470 km s^{-1} at 100 kpc. From 10–100 kpc, the data indicate that the velocity dispersion is fairly constant at ~450 km s^{-1}, but Dressler notes that his measurements at 10 kpc may be affected by light from a superposed galaxy. Dressler's surface photometry indicates that the volume luminosity density $\ell(r)$ in this galaxy decreases as a power law $\ell \sim r^{-2.37}$ over the range 10 kpc $< r <$ 100 kpc, with the result that if the velocity dispersion is assumed to be isotropic and constant over this range, Equation (7) indicates that M/L rises as $\sim r^{0.37}$ between 10 kpc and 100 kpc, or by a factor of 2.3. In fact, Dressler's conclusion that in this galaxy M/L increases outward seems secure so long as the velocity dispersion tensor there is not strongly anisotropic Carter et al. (1981) find that the velocity dispersion in IC 2082 rises from 260 km s^{-1} at the center to 300 km s^{-1} at about 23 kpc. IC 2082, which lies toward the center of a Bautz-Morgan type I–II cluster, has a rather complex structure involving a faint nucleus ~8 kpc from the main nucleus of the galaxy. In view of this substructure and the absence

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of complete photometry, it is difficult to arrive at a secure interpretation of the measurements of Carter et al. However, the same arguments that led Dressler to conclude that M/L increases toward the outside of the central galaxy in Abell 2029, suggest that IC 2082 also has variable M/L.

In normal giant elliptical galaxies the velocity dispersion is not observed to increase at large radii (see Figure 9 of Davies 1981). In some galaxies, for example NGC 3379 and NGC 4472, the velocity dispersion decreases with increasing radius as rapidly as the simplest spherical stellar models predict. In other galaxies, notably NGC 4697, the velocity dispersion does not diminish outward so rapidly. It is still unclear whether these variations in the behavior of σ_v with r may be understood in terms of models that have constant M/L. Binney (1980b) has shown that variations in the brightness profiles of elliptical galaxies lead to interesting variations in the $\sigma(r)$ profiles of these galaxies, even when M/L is assumed constant and the velocity dispersion tensor isotropic. But it is possible that observations of some normal elliptical galaxies may not be compatible with constant M/L at large radii.

4. NONSPHERICAL SYSTEMS

4.1 *Models*

For many years it was taken for granted (e.g. Sandage 1964) that galaxies are axisymmetric oblate bodies. Recently this assumption has been questioned by a number of workers for reasons that are part observational and part theoretical (Stark 1977, Williams & Schwarzschild 1979, Binney 1978b, Miller & Smith 1980). At present, it would be unwise to rule out the possibility that all elliptical galaxies are oblate and axisymmetric, but this now seems very unlikely. In this subsection, I review the theoretical situation as regards both axisymmetric and triaxial systems. The axisymmetric models are likely to be good guides to the structure of nearly axisymmetric triaxial bodies in the same way that spherical systems help us to understand the radial structure of mildly nonspherical galaxies, and they may well have direct application as models of any truly axisymmetric spheroidal components.

AXISYMMETRIC MODELS The energy E and the component of angular momentum about the symmetry axis of the system J_z are always integrals of the motion for a star orbiting in an axisymmetric galaxy, and the simplest models of nonspherical galaxies have distribution functions that depend only on E and J_z. These models are special in that everywhere within them the velocity dispersion in the radial direction equals that parallel to the symmetry axis (i.e. $\langle v_R^2 \rangle = \langle v_z^2 \rangle$). This is unfortunate because in the solar neighborhood, stars that belong to the galactic halo, for example the halo RR Lyrae stars, have $\langle v_R^2 \rangle \sim 4 \langle v_z^2 \rangle$ (Woolley 1978). However, there may be some systems,

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for example globular clusters, to which these models do apply, and it is useful to consider the structure of the simplest axisymmetric models before passing on to more general (and much more intractable) models.

Models from distribution functions Prendergast & Tomer (1970; henceforth referred to as PT) pioneered the study of models based on $f(E,J_z)$ by constructing models based on distribution functions of the form

$$f(E,J_z) = f_{\rm I}(E)\exp(\Omega J_z/\sigma^2) \qquad E < E_{\rm t}$$
$$= 0 \qquad E \geq E_{\rm t}, \tag{21}$$

where $f_{\rm I}$ is defined by Equation (9). Near the center of these models the rotation speed $\langle v_\phi \rangle$ rises as $\langle v_\phi \rangle = \Omega R$ and is constant on cylinders. At points that are more than halfway to the tidal surface, the rotation speed declines with increasing radius and, in fact, vanishes at the tidal surface itself. In the outer portion the rotation speed is approximately constant on spheroids.

The flattening of the isodensity surfaces in the PT models reflects the shape of the rotation curve. At the center and near the tidal surface, the isodensity surfaces tend to be round, so that the galaxy is strongly flattened only near the peak in the rotation curve. Hunter (1977) has shown that any distribution function that is the product of a function of E and a function of J_z must generate a model whose isodensity surfaces become spherical at the center.

Wilson (1975) used an improved version of the computational method introduced by PT to construct models whose distribution functions are of the form

$$f(E,J_z) = f_{\rm W}(E)\exp[\Omega J_z/\sigma^2 - \tfrac{1}{2}a^2(\Omega J_z/\sigma^2)^2] \qquad (E < E_{\rm t}), \tag{22}$$

where $f_{\rm W}$ is given by Equation (14) and a is an additional parameter. When $a \neq 0$, this distribution function differs from that of PT only in that its energy dependence is that of a Wilson sphere rather than that of a Woolley model. The general characteristics of these models are very similar to those of the PT models. The parameter a in Equation (22) causes the rotation curve to flatten off at a radius that is independent of the tidal radius. Unfortunately, when $a \neq 0$ the outer part of the model becomes elongated along the symmetry axis, which is rather unphysical. For this reason Wilson considered only values of a at which the second term in the exponent of Equation (22) was comparatively unimportant.

It is a pity that a distribution function has not been devised that generates models with isodensity surfaces whose ellipticity is nearly independent of radius and whose rotation curves are correspondingly flat. This is because the observations of early-type disk galaxies (see below) indicate that the spheroidal components of these galaxies have such characteristics. A device that was introduced by Lynden-Bell (1962a) and improved by Hunter (1975) might

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be employed to guide one in the choice of a more appropriate distribution function. Lynden-Bell showed how, given the density distribution of a galaxy having $f(E,J_z)$, one may recover the part f_+ of f that is even in J_z. The part f_- that is odd in J_z cannot be recovered from $\rho(R,z)$, but it is possible that it is determined by $<v_\phi>(R,z)$. In either of its present forms, Lynden-Bell's technique requires that $\rho(R,z)$ be extended to a complex analytic function of R and Φ, which makes the method difficult to formulate numerically and has until now restricted the technique to rather unrealistic density distributions.

An interesting application of Lynden-Bell's device is found in the work of Lake (1981a,b) on prolate galaxies. Lake first recovered the $f_+(E,J_z)$ that generates a type of prolate Plummer model. In these models the density is infinite on the symmetry axis of the system, and this is reflected in f_+ containing a Dirac delta-function of J_z. Lake modified f_+ by smearing the delta-function out into a peak of finite width, and then solved for the density structure associated with the modified form of f_+ using the methods employed by PT. In this way he obtained a prolate model whose density is nowhere infinite. This model is probably of only academic interest, since prolate galaxies are very unlikely to have $<v_R^2> = <v_z^2>$, but it is a nice illustration of how Lynden-Bell's device may be profitably employed in the future.

The problem of constructing general axisymmetric models in which $f=f(E,J_z)$ and $<v_R^2> \neq <v_z^2>$, as is required by observations of the galactic halo, will normally take one beyond the range of validity of Jeans' theorem because it requires that one take into account the complexities to which the "third integral" gives rise. As indicated in Section 2, many (perhaps most) orbits in a reasonably smooth potential respect three isolating integrals, which in an axisymmetric potential we may denote E, J_z and I_3. If one knew the analytic form of $I_3(\mathbf{x},\mathbf{v};\Phi)$, one might construct models by treating it on a par with E and J_z (Lynden-Bell 1962b). But a simple expression does not exist for I_3, even for the regular orbits, and Jeans' theorem breaks down soon when orbits exist that are neither ergodic nor regular.

Schwarzschild (1979) has developed a technique for constructing self-consistent models around a given density distribution that works even in the presence of irregular orbits. He follows a large number of orbits in the potential associated with his chosen density distribution and then uses a linear-programming technique to populate a selection of these orbits. This is done in such a way that the time-averaged density contributed by these orbits to each of a large number of cells throughout the system equals the density originally assumed. Schwarzschild developed this technique to handle the difficult problem of constructing triaxial galaxies, but it is eminently well suited to the construction of general axisymmetric galaxies. Richstone (1980, 1982) has employed a variant of this method to build a special type of axisymmetric galaxy—that in which the isodensity surfaces are similar spheroids and

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$\rho \sim R^{-2}$. It is to be hoped that more general models of this type will be constructed in the near future.

Models from moment equations The next best thing to a solution of the coupled Vlasov and Poisson equations is information about the relationships that hold between the velocity moments of a galaxy of given density distribution. Satoh (1980) used the Jeans equations to derive the velocity dispersion $\sigma(R,z)$ and the rotation speed $<v_\phi>(R,z)$ of a galaxy of given density on the assumption that the velocity dispersion in the system is everywhere isotropic. The density distributions studied by Satoh are modified Plummer models in which $\rho \sim R^{-3}$ at large R in the equatorial plane, and $\rho \sim z^{-6}$ far out along the symmetry axis. The curves of $<v_\phi>(R,0)$ peak many core radii from the center and then flatten off until the circular velocity has fallen to a value nearly equal to $<v_\phi>(R,0)$. The velocity dispersion in these models falls steeply with increasing radius.

Binney (1980b) has applied the tensor virial theorem to the volumes of systems that are bounded by isodensity surfaces to estimate $<v_\phi>(R,0)$, using a number of assumptions about the way in which $<v_\phi>$ varies with R and z and the degree of anisotropy of the velocity dispersion tensor. The systems studied included model galaxies in which $\rho \sim (R^2 + z^2/q^2)^{-3/2}$ at large R and z (q being the axial ratio), and models of three of the galaxies studied photometrically by King (1978). These models suggest that one cannot depress $<v_\phi>$ to the degree that is required by the observations (see below) unless the velocity ellipsoids have principal axes that near the center align with the equatorial plane, rather than with the radial direction. The rotation curves that King's galaxies would require if their velocity dispersion tensors were isotropic rise very steeply near the center and then become remarkably flat. The velocity dispersion in these galaxies should decline slowly with increasing radius.

N-body models N-body models are superior even to exact solutions of the Vlasov equation in one important respect: they are easily tested for stability. Against this signal advantage must be set their cumbersomeness and the difficulty encountered by older programs in handling both large numbers of particles and large density contrasts. Fortunately, algorithms have now been developed by (among others) van Albada (1982), Villumsen (1982), and McGlynn & Ostriker (in preparation) that are able to combine the flexibility as regards density contrasts of programs that calculate forces by direct summation over all particles (e.g. Ahmad & Cohen 1973) with the ability of the Fourier approach to the force calculation to handle enormous numbers of particles.

One makes a galaxy model with an N-body code by allowing particles to relax to a steady state from some initial configuration. The initial conditions

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generally are a homogeneous distribution within some boundary, with a Gaussian velocity dispersion superposed on some degree of rigid-body rotation. However, as Gott (1975) has emphasized, galaxies are unlikely to have relaxed from homogeneous initial configurations. This is unfortunate since the numerical experiments show that the initial conditions from which relaxation occurs influence the final state.

Gott (1973), Gott & Thuan (1976), Miller (1978), Miller & Smith (1979), Hohl & Zang (1979), van Albada (1982), and others have studied the collapse of rotating spheres of stars, while Binney (1976), Aarseth & Binney (1978), and Miller & Smith (1981) have studied the collapse of initially flattened stellar distributions. These investigations show the following. (*a*) Systems formed by relaxation from homogeneous initial configurations are as centrally concentrated as are galaxies only if the initial conditions are very cold. (*b*) In the absence of dynamically significant rotation of the initial state, initially spherical systems relax to spheres and initially flattened systems relax to spheroids. (*c*) The velocity ellipsoids of relaxed systems have a strong radial bias in the outer regions, and become in the almost homogeneous core either isotropic (if the system is spherical) or oblate and aligned with the figure of the system (if the system as a whole is flattened). In the latter case, $f \neq f(E, J_z)$. (*d*) Rapidly rotating initial configurations form tumbling bars rather than highly flattened axisymmetric bodies. Hohl & Zang (1979) find that the flattest axisymmetric body that can be formed from a rotating homogeneous stellar sphere is E2. Axisymmetric bodies of flattening as high as E7 can be formed by the relaxation of flattened nonrotating initial configurations.

These results indicate that phase mixing and violent relaxation (Lynden-Bell 1967) work too inefficiently to be capable of imposing a uniform stamp on elliptical galaxies. However, the collapse calculations show that plausible initial conditions will lead to the formation of systems that closely resemble the galactic halo and nearby elliptical galaxies.

TRIAXIAL MODELS Collapse calculations of the type just described suggest that triaxiality is likely to be common whether the initial configuration is one of rapid or slow rotation. Indeed if the initial configuration is not axially symmetric and the rotation of the initial state is dynamically unimportant, it is hard to see what might determine a particular body axis as an axis of symmetry. Aarseth & Binney (1978) and Wilkinson & James (1982) have verified that systems that relax from slowly or nonrotating triaxial configurations do form triaxial galaxies that appear to be long-lived (but see Sanders & van Albada 1979). If, however, the initial state is one of rapid rotation, the system is found to form a tumbling bar that has no axis of symmetry (Miller & Smith 1979, Hohl & Zang 1979).

While it is easy to form individual bars with an *N*-body program, it is difficult to isolate the general principles of bar dynamics. Schwarzschild's

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construction of stellar bars from individual orbits in a given potential (1979 and work in progress) helps one come closer to these general principles. This work and that of Wilkinson & James (1982) indicate that the backbone of a slowly rotating stellar bar is a population of stars on orbits that may be considered epicyclic developments of the closed orbit that lies along the long axis of the potential. These orbits may or may not have a definite sense of circulation about the long axis of the system, but they have a definite sense of circulation about either of the shorter axes of the system only if the figure of the potential rotates. Then they circulate with respect to the figure in the same sense as that of the figure with respect to inertial space. This suggests, and the available *N*-body models confirm, that there is a fairly tight connection between the pattern speed with which the figure of the bar rotates and the speed with which the stars stream with respect to the figure; bars that have large pattern speeds show strong streaming motions with respect to the pattern of the bar. In particular, it should be possible (in principle) to estimate the pattern speed of a bar from a knowledge of the magnitude of the overall circulation and an estimate of the axial ratios of the bar. Conversely, if one can argue that the pattern speed of a particular galaxy must be small, because its brightness profile shows no sign of a characteristic radius where corotation or a Lindblad resonance may occur, it may be possible to use the magnitude of the stellar rotation velocity close to the center of the galaxy to limit the deviation of the figure of the galaxy from axial symmetry.

Resonances are bound to play an important role in the dynamics of triaxial elliptical galaxies, and one would certainly expect the brightness profiles of such galaxies to show features at the characteristic radii of such resonances. It is possibly worth recalling in this connection the elliptical rings that have been found by Malin & Carter (1980) on high-contrast prints of elliptical galaxies.

4.2 *Observations of Nonspherical Structure*

PHOTOMETRY A number of authors have studied the shapes of the isophotes of elliptical galaxies during the last five years (King 1978, Carter 1979, Bertola & Galletta 1979, di Tullio 1979, Williams & Schwarzschild 1979, Leach 1981). The key points to have emerged from this work are the following.

1. The isophotes of ellipticals show little or no deviation from pure ellipses. This contrasts with the box-like shapes of the bulges of some lenticular galaxies.

2. The ellipticity $\epsilon(r)$ can vary with semimajor axis length r in a complex way. The isophotes generally do not tend to become circular near the center as models that have distribution functions of the form $f = g(E)h(J_z)$ require, though di Tullio (1979) finds that this type of behavior is characteristic of the

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brightest galaxies in clusters and groups. The isophotes of isolated galaxies usually become more elliptical as the center is approached, and every type of variation of $\epsilon(r)$ occurs in general group and cluster members.

3. In some galaxies the major axes of different isophotes are not parallel to one another—that is, the isophotes are twisted. This phenomenon would not be possible if elliptical galaxies were axisymmetric and dust-free. It is readily understood if the isodensity surfaces of ellipticals are coaxial but triaxial ellipsoids, for then changes in the axial ratios of these ellipsoids would lead to the isophotes twisting on the sky (e.g. Mihalas & Binney 1981). Alternative explanations are that the isodensity surfaces are not coaxial, or that absorption by dust substantially alters the brightness profiles of these systems. Neither of these seems particularly attractive.

Statistical studies of the frequency with which elliptical galaxies of various apparent axial ratios occur have shown that the data may be accounted for very satisfactorily under any hypothesis as to the true shapes of elliptical galaxies: the galaxies may be oblate or prolate figures of revolution or any of the triaxial ellipsoids that lie between these extremes (Binney 1978a, Noerdlinger 1979, Binggelli 1980, Binney & de Vaucouleurs 1981). Under any hypothesis, the commonest ratio of the shortest to the longest body axes lies near 0.6:1. If most ellipticals are prolate, there must be more nearly spherical galaxies than if the oblate type is commonest.

If elliptical galaxies were identical prolate bodies, the central surface brightness of the best-fitting $r^{1/4}$ profile of an apparently round galaxy would be higher than the corresponding quantity for an apparently highly elongated galaxy—and vice versa if galaxies were all oblate bodies. Attempts to detect such a correlation between surface brightness and apparent ellipticity (Marchant & Olson 1979, Richstone 1979) have tentatively concluded that the galaxies are more likely to be oblate than prolate. Lake (1979) has discussed the possibility that there is a similar correlation between ellipticity and central line-of-sight velocity dispersion. He concludes from a small data set that the galaxies studied may be prolate.

Galletta (1980) has confirmed that the most pronounced isophotal twists tend to occur in E1 and E0 galaxies. Only a small shift in the relative lengths of the body axes of successive isodensity surfaces in a nearly spherical galaxy is required to interchange the major and the minor axes and thus swing the apparent major axis through 90°.

The relative orientation of optical galaxies and any associated gas, dust, and radio components may lead to inferences about the shape of the optical galaxy (Bertola & Galletta 1978). Gas clouds are expected to settle quickly into closed orbits (Tohline et al. 1981). At most radii the only suitable closed orbits circulate about either the longest or shortest body axes of the potential (Heiligman & Schwarzschild 1979, Binney 1981); thus the plane of any dust or gas

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disk may be assumed to define one of these fundamental planes. Furthermore, observation suggests (Kotanyi & Ekers 1979) that radio jets tend to emerge along the rotation axes of these disks, just as one might expect if the radio sources are powered by accretion of the material of the disks onto compact objects. If radio axes are good tracers of the body axes of ellipticals, it is interesting that Battistini et al. (1980) find no evidence that radio axes align with the apparent principal axes of a sample of 51 galaxies, for this would indicate that these (abnormal) galaxies are strongly triaxial. Van Albada et al. (1981) have discussed the possibility of determining the angular velocity of the figures of radio galaxies from their radio structure.

SPECTROSCOPY The mean stellar rotation velocity has now been measured along the major axis of a few dozen elliptical galaxies out to radii $r > 2$ kpc at which the rotation curves cease to rise steeply. Some rotation is normally detected. By contrast, attempts to detect streaming motions along the apparent minor axes of normal galaxies have not yet produced unambiguous evidence of minor-axis rotation (Williams 1979, Schechter & Gunn 1979). However, Jenkins & Scheuer (1980) find that two of three radio galaxies show minor-axis rotation.

The major-axis rotation curves are generally characterized by a steep rise just outside the center, followed by a long, flat or slightly falling portion. Exceptions to this pattern do occur, however. For example, Efstathiou et al. (1980) find that the rotation speed of the E1 galaxy NGC 5813 peaks 5″ from the nucleus and then drops steeply to zero.

It is convenient to discuss the rotation of spheroidal components in terms of the relationship between a characteristic ellipticity ϵ and the ratio v_p/σ of the peak line-of-sight rotation speed v_p to the central velocity dispersion σ. Figure 1 plots the relationship between these variables for elliptical galaxies. Evidently ϵ is not correlated with v_p/σ.

This result enables one to eliminate immediately the simplest picture of an elliptical galaxy—that in which the galaxy is an oblate spheroid in which the velocity dispersion is everywhere fairly isotropic and rotational-streaming is responsible for the galaxy's flattening [the isotropic oblate (IO) picture]. In fact, the flat rotation and velocity dispersion profiles of ellipticals imply that the projected central velocity dispersion should be nearly equal to the rms velocity dispersion through the entire galaxy; in addition, the projected peak rotation velocity in the IO picture should differ from the rms rotation velocity by a factor that depends in a simple way on the inclination angle i between the symmetry axis of the system and the line of sight. On the other hand, in this picture the apparent axial ratio of the galaxy also depends on the inclination i in a simple way, so that as i varies, the representative point of the galaxy in Figure 1 will move along a well-defined path. By good fortune, in the IO

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picture this path nearly coincides with the curve formed by the representative points, as estimated from the tensor virial theorem (Binney 1978a) of IO models that have $i = 90°$ and various true flattenings. This curve is marked IO in Figure 1. If elliptical galaxies were rotationally flattened oblate figures of rotation, their representative points would lie close to these nearly coincident lines, rather than scattered all over the lower portion of the diagram.

It has been suggested that despite the wide scatter of the observational points in Figure 1, the true shapes of elliptical galaxies may be due to rotation, but the rotation of prolate bodies that tumble end over end (Miller & Smith 1980). It is easy to understand that if elliptical galaxies were tumbling bars, there could be no tight correlation between ϵ and v_p/σ; when such a bar is viewed down its rotation axis it has large ϵ and small v_p/σ, and just the reverse when it is viewed down its long axis. In fact, the representative points of a single tumbling bar would occupy a more or less triangular region, two of whose sides run parallel to the axes in Figure 1, depending on the orientation of the bar to the line of sight. Hence in this picture one would expect the representative points of a number of galaxies to lie close to the v_p/σ axis in Figure 1 (where in fact there are no points). Furthermore, if one assumes that the

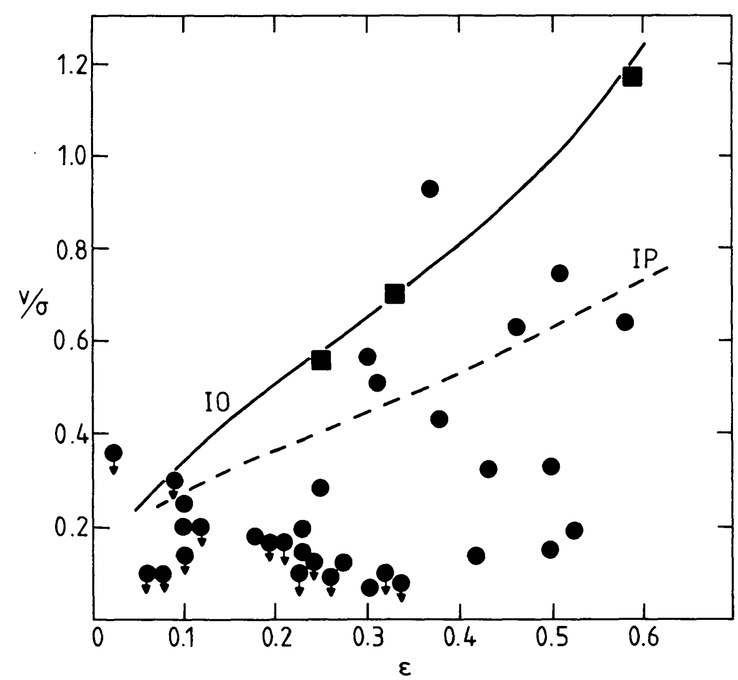

Figure 1 The ratio v_p/σ of the maximum line-of-sight rotation velocity to the mean velocity dispersion is plotted against ellipticity ϵ for 33 elliptical galaxies (circles). The full line shows the relationship between v_p/σ and ϵ that is predicted by the tensor virial theorem in the isotropic oblate (IO) picture. If the galaxies conform to the isotropic prolate (IP) picture, half the representative points would lie above the broken line. The squares mark the representative points of the bulges of three disk galaxies—NGC 3115, M31, and M81. [Adapted from Illingworth & Schechter (1981) by permission].

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velocity dispersion in these bars is always isotropic, one may again apply the virial theorem and the distribution of true ellipticities that are required of such bars if the predicted distribution of apparent ellipticities is to match the observed distribution, to predict the expected distribution of points in Figure 1. The line in the figure marked IP is such that half of the points should lie above this line. Evidently elliptical galaxies cannot be prolate bars having isotropic velocity distributions.

Recent work on the bulges of early-type disk galaxies indicates that these systems are quite unlike ellipticals, in that rotation probably accounts for their flattenings. Kormendy (1981a) and Illingworth and co-workers (Kormendy & Illingworth 1982, Illingworth & Schechter 1981, Verter et al., in preparation) have mapped the velocity fields of a few normal lenticular and spiral galaxies. When they extract from their observations the probable rotation patterns of the bulges of these systems, they find (*a*) the ratio of rotational to random kinetic energy in these systems is nearly equal to that expected in the IO picture (see Figure 1); (*b*) the shapes of the rotational curves of these galaxies are similar to those of giant ellipticals; near the center $\langle v_\phi \rangle$ rises steeply and then becomes constant; and (*c*) most of these systems rotate as if they were constructed of rigid coaxial spheroids, although one galaxy, NGC 4565, which has a box-shaped bulge, rotates on cylinders.

Kormendy (1981b) has studied the kinematics of six barred lenticulars. He finds that the bulges of these galaxies rotate at least as rapidly as the IO picture would suggest. Some of the bulges that appear to be triaxial rotate even faster than in the IO picture. This is to be expected if these bulges are tumbling bars that have near-isotropic velocity dispersion tensors.

Work undertaken by Davies et al. (in preparation) suggests that the differences between the rotation properties of giant ellipticals and those of the bulges of disk galaxies are related to differences in the luminosities of these two types of system; the absolute magnitudes of the bulges that contribute to Figure 1 lie in the range $-18 > M_B > -21$, whereas the great majority of the elliptical galaxies contributing to this figure have $M_B < -20.5$. Davies et al. find that all very-low-luminosity ellipticals rotate as rapidly as bulges of the same luminosity. Why this should be so is not clear.

5. SUMMARY AND PROSPECTS

Despite rapid progress in recent years, our understanding of spheroidal components remains imperfect. Some of the more important questions that have to be answered are the following.

1. What is the structure of the nuclei of spheroidal systems? How common are cusps of brightness at the centers of galaxies? Are the structures of these cusps affected by massive black holes at their foci? What relationship have

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these stellar structures to active galactic nuclei and quasars? High-resolution, high signal-to-noise photometry and further work on the dynamics of systems whose distribution functions are of the form $f(E,J)$ are required to elucidate these questions.

2. Do normal giant elliptical galaxies have massive halos? The rotation curves of disk galaxies tell us that the mass distributions of these galaxies are more extensive than the light distributions of their bulges. Dynamical arguments (Ostriker & Peebles 1973, Efstathiou et al. 1982) suggest that this additional mass is not associated with the disks, so we may conclude that these bulges have massive halos. Furthermore, radial velocities of cluster galaxies suggest that the mean mass-to-light ratio near the centers of rich clusters is very much higher ($M/L_B \gtrsim 200$; Faber & Gallagher 1979) than the mass-to-light ratios ($M/L_B \lesssim 10$) that are indicated by the internal motions of the galaxies. Since most of the light from the central region of a compact rich cluster (such as that in Coma) comes from spheroidal components, it is probable that these spheroidal components either possess or have possessed massive halos. However, the uncertainties associated with interpreting velocity dispersion profiles in dynamical terms, and the difficulty of pushing spectroscopic observations to very low surface brightnesses, are such that it has not been demonstrated that any normal elliptical galaxy has a massive halo. Dressler's (1979) work on the cD galaxy in the cluster Abell 2029 presents the strongest case for a massive halo around an elliptical galaxy.

3. Are spheroidal components normally triaxial? Further observations of gas disks and dust lanes within and around elliptical galaxies should help us to elucidate this fundamental question, as would a better understanding of the dynamics of triaxial systems. Do the apparent major axes of the bulges of many disk galaxies run, like that of the bulge of M31, not exactly parallel to the major axis of the surrounding disk? If spheroidal components are triaxial, are they predominantly oblate or prolate?

4. What are the pattern speeds of triaxial spheroidal components, and how do these speeds relate to the observed streaming motions within elliptical galaxies and the bulges of disk galaxies? It has yet to be demonstrated that triaxial systems are possible that (like giant elliptical galaxies) have nearly singular central densities and rather flat rotation curves, and yet have no obvious features in their density profiles at the characteristic radii of resonances. The very nearly featureless brightness profiles of elliptical galaxies suggest that the speeds at which their figures rotate must be very small, and it is not clear that such small pattern speeds will permit appreciable rotation very close to the centers. One possibility is that spheroidal components become axisymmetric oblate bodies at a radius that is smaller than the radius of the first important resonance.

5. Why do the bulges of disk galaxies rotate rapidly and giant elliptical

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galaxies slowly? Does this difference in rotation speeds reflect only the different luminosities of the two types of system? Can one account in detail for the observations of bulges with models in which the velocity dispersion tensor is isotropic?

It may be many years before we possess a satisfactory understanding of the shapes and internal motions of most spheroidal components. The two most urgent theoretical tasks involve the construction of axisymmetric models, which remain of great interest since many spheroidal components may turn out to be axisymmetric, and the structure of axisymmetric models is anyway likely to help us understand nearly axisymmetric triaxial galaxies. Two types of axisymmetric models are urgently needed. Recent observations of the bulges of disk galaxies and of dwarf ellipticals call for models based on distribution functions $f(E,J_z)$. The problem here is choosing a distribution function that generates a model whose isodensity surfaces are nearly similar ellipsoids and whose rotation curve is rather flat. Observations of giant elliptical galaxies and of the kinematics of stars in the solar neighborhood call for more general models in which $<v_R^2> \neq <v_z^2>$ and the distribution function is not a function of just the classical integrals E and J_z.

In the observational area the greatest need is now for high-quality surface photometry of all types of spheroidal components. Photometric observations are every bit as vital as spectroscopic measurements for developing our understanding of the dynamics of spheroidal components, and there is a strong case for correcting the imbalance that has arisen in the allocation of scarce telescope time between spectroscopic and photometric observations.

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