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Thermal Phases of the Interstellar Medium in Galaxies

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Abstract. This review deals with the theory of multiphase media in astrophysical systems. I discuss the basic reasons for the existence of multiple thermal phases, and the fundamental connection between multiphase media and thermal instability. After describing important examples of multiphase media, I examine the interactions among phases, i.e., mass exchange driven by thermal conduction and hydrodynamic ablation. Mass exchange may compete with radiative heating and cooling for control of the thermal state of the hot phase, and may alter the thermal stability properties of the system.

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1. INTROUCTION

Astrophysical gases are often highly inhomogeneous, with two or more "thermal phases" coexisting in rough pressure balance with one another. Compared to the scales of typical inhomogeneities, the transitions between neighboring regions of different temperature (and density) can be quite sharp. Such systems are most often modeled as consisting of cold clouds, filaments or sheets embedded in a hotter intercloud medium, although there may be cases in which a model consisting of hot bubbles in a cold matrix is more appropriate. Usually the topology of the phases is highly uncertain, but the conditions which lead to their existence are more reliably established. The temperatures of the phases sometimes differ by orders of magnitude, and are frequently set within rather narrow ranges by the details of atomic and molecular processes or by the spectrum of ambient radiation. It should be stressed that thermal pressure balance may not be exact, e.g., where magnetic fields or cosmic rays supply a significant fraction of the pressure in one or more phases, where self-gravity or turbulent pressure are dynamically important, where ram pressure (associated with differential motion of the phases) provides part of the confinement, in the case of a cool cloud evaporating suprathermally in a hot background (Balbus and McKee 1982), or when there is simply too little time for a system to achieve dynamical equilibrium. Although the concepts of multiphase media are not generally used to describe regions which are wildly out of dynamical equilibrium with their surroundings (such as material behind a propagating shock front), localized pressure fluctuations may be an important means of transferring material between phases (Wang and Cowie 1988).

This review presents an overview of the theory of thermal phases, with particular attention to their role in the interstellar media of galaxies. In [Section 2](#) I discuss the basic reasons for the existence of multiphase media, and show the connection between multiple phases and thermal instability. I also give examples of multiphase systems which are important in astrophysics. Since

the phases are in physical contact, it is unrealistic to treat them as being isolated from one another. [Section 3](#) deals with the principal interactions among phases, thermal conduction and ablation. Taking these interactions into account is particularly important if one wishes to understand the temporal evolution of multiphase media; [Section 4](#) deals with the consequences of mass exchange and with evolutionary models. The "state of the art" is summarized in [Section 5](#). Much of the original material presented in this review was developed in collaboration with C. F. McKee, and is described in greater detail in Begelman and McKee (1990).

2. THEORY OF MULTIPHASE MEDIA

2.1. Why thermal phases?

The existence of multiple thermal phases is made possible by the flow of energy into and out of a system. Multiple phases do not develop in systems which are thermodynamically isolated from their surroundings. If $\Gamma(n, T, x_j)$ is the heating rate per particle and $\Lambda(n, T, x_j)$ is the cooling function, then the equation of thermal equilibrium may be written

$$n^2\Lambda - n\Gamma \equiv n^2\mathcal{L} = 0, \quad (1)$$

where n is the density of hydrogen nuclei, T is the temperature, and x_j represents the fractional concentrations of various species, $x_j \equiv n_j/n$. The pressure is given by $p = x_t nkT$, where $x_t = \sum x_j$ is the number of particles per hydrogen nucleus. An equation analogous to (1) determines the ionization equilibrium. Γ , and sometimes Λ (e.g., in the case of inverse Compton cooling), may also depend on the magnitude of some external heating or ionization agent, which has energy density $u\Gamma$. If $u\Gamma$ is held fixed then the solution of the equilibrium equations generates a curve in the $p - n$, $p - V$ (where $V \equiv 1/n$ is the specific density) or $p - T$ plane which separates the heating region ($\Gamma > n\Lambda$) from the cooling region ($\Gamma < n\Lambda$). In general these curves may have complex shapes and be multivalued.

If there are two or more values of n (or, equivalently, of T), which correspond to a given pressure, then a *multiphase equilibrium* is possible: a relatively cool, dense region can coexist with one or more warmer, less dense regions in pressure equilibrium. If this configuration is thermally stable (see [Section 2.2](#) below), and if there is no mass exchange between phases, then this equilibrium can persist indefinitely. Simple generic examples of multiphase equilibria are shown in [Figure 1](#). In all cases we have assumed that there is a single stable "cloud" phase with a fixed temperature T_{cl} . Figs. 1 *b* and 1 *d* both show systems with two stable phases, while the other panels show systems with only one stable phase. More realistic phase diagrams may show three or more stable phases, e.g., Lepp et al. (1985).

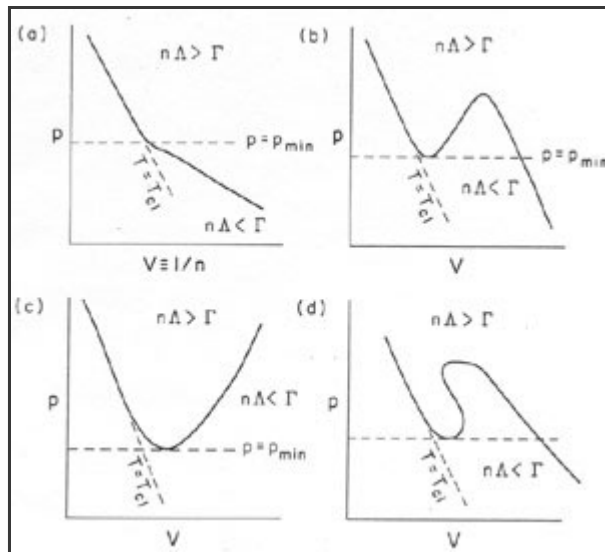


Figure 1. Radiative thermal equilibrium in the $p - V$ plane. Cooling exceeds heating above the solid line. Clouds exist for pressures exceeding p_{\min} their temperature is fixed at T_{cl} . a) One thermally stable phase. b) Two thermally stable phases separated by a thermally unstable phase. c) Thermally stable clouds in a thermally unstable intercloud medium. d) If $p(V)$ is multivalued (corresponding to gas at a given density having one of several temperatures), the gas may be isochorically as well as isobarically unstable.

In most cases of astrophysical interest, Γ is linear in $u\Gamma$. If both Λ and Γ are independent of density (as in particle or photon heating and two-body cooling) or depend on density through the ratio $n/u\Gamma$ (as in the case of inverse Compton cooling), then both the ionization level and the temperature depend on n and $u\Gamma$ only through the combination $n/u\Gamma$ or, equivalently, $p/u\Gamma$. This similarity variable is very useful for characterizing the state of gas heated by cosmic rays (Dalgarno and McCray 1972) or radiation (e.g., Tarter, Tucker and Salpeter 1969; Davidson 1972; Krolik, McKee and Tarter 1981); in various forms it is referred to as the ionization parameter.

Not all thermal phases which are observed in astrophysical systems correspond to stable equilibria. Examples of systems which exhibit long-lived non-equilibrium hot phases, in pressure balance with a stable cold phase, are the three-phase interstellar medium (McKee and Ostriker 1977) and cooling flows in clusters of galaxies (Sarazin 1986, and references therein). The non-equilibrium phases in these systems are in fact thermally unstable, and are observable only because their cooling time scales are extremely long (Spitzer 1956). An accurate analysis of such systems requires the treatment of time-dependence (McKee and Ostriker 1977) and hydrodynamical effects such as buoyancy (Balbus and Soker 1989).

2.2. Connection with thermal instability

There is an intimate connection between the existence of thermal phases and the thermal stability of a system: any system exhibiting multiphase equilibria must be thermally unstable over a range of thermodynamic parameters. The thermal stability of astrophysical gases was first studied systematically by Field (1965). His instability criterion was generalized to non-equilibrium systems by Balbus (1986 *a*), who found the following condition for instability:

$$\left[\frac{\partial(n\mathcal{L}/T)}{\partial s} \right]_A < 0. \quad (2)$$

Here s is the entropy per hydrogen nucleus and A is some thermodynamic variable which is held constant during the perturbation. In equilibrium, $\mathcal{L} = 0$ and this reduces to Field's instability criterion

$$\left(\frac{\partial \mathcal{L}}{\partial s}\right)_A < 0 \quad (\mathcal{L} = 0). \quad (3)$$

In general, s is a complicated function of n , T , and the state of ionization of the gas. However, in many applications the gas is almost completely ionized and the entropy function may be approximated by the expression for an ideal gas, $s \sim \ln p V^{5/3} + \text{const}$. If A is some power law combination of p and V , then $T(\partial s / \partial T)_A$ is a constant specific heat which is positive for cases of interest. The instability criterion then becomes

$$\left[\frac{\partial(n\mathcal{L}/T)}{\partial T}\right]_A < 0. \quad (4)$$

Since the cooling time is proportional to $T / n \mathcal{L}$, this criterion can be rephrased as stating that instability occurs if the cooling time increases with temperature (Balbus 1986a).

If the gas is in equilibrium ($\mathcal{L} = 0$), the instability criterion (4) reduces to

$$\left(\frac{\partial \mathcal{L}}{\partial T}\right)_A < 0 \quad (\mathcal{L} = 0). \quad (5)$$

Field (1965) showed that for the equilibrium case the isobaric criterion $(\partial \mathcal{L} / \partial T)_p < 0$ is usually the correct one to apply. However, if the system is large enough that the sound crossing time is long compared to the heating or cooling times, then for long wavelengths the isochoric criterion $(\partial \mathcal{L} / \partial T)_V < 0$ is applicable.

The stability criterion (5) may be interpreted geometrically in terms of the equilibrium curve (Figure 1). Typically the cooling region ($n \Lambda > \Gamma$) lies above the heating region because the cooling rate usually increases faster with n and T than does the heating rate. If, on the other hand, the heating region lay above the cooling region, then over much of the curve (wherever $p(V)$ is single-valued) one would have $(\partial \mathcal{L} / \partial T)_V < 0$ and the equilibrium would be isochorically unstable. In this case systems large enough that the sound crossing time is much greater than the heating and cooling times could be unstable even where smaller systems are isobarically stable. This situation does not arise in practice and we therefore assume that the cooling region lies above the heating region in the $p - V$ plane, as shown in Figure 1.

The slope of the equilibrium curve in the $p - V$ plane is directly related to the stability of the system since

$$\left(\frac{dp}{dV}\right)_{\mathcal{L}=0} = - \left(\frac{\partial \mathcal{L}}{\partial T}\right)_p / \left(\frac{\partial \mathcal{L}}{\partial T}\right)_V \quad (6)$$

(Field 1965). For cases in which $p(V)$ is single valued (as in Fig. 1 a-c), the condition that the cooling region lie above the heating region implies that the system is isochorically stable, so that the denominator in equation (6) is positive; hence, in this case isobarically stable regions have a negative slope in the $p - V$ plane, whereas unstable regions have a positive slope. The condition for a multiphase equilibrium is that $V(p)$ be a multivalued function, which is equivalent to having $d \ln p / d \ln V$ change sign. Thus, a necessary and sufficient condition for the existence of a multiphase equilibrium is that the system be thermally unstable over a finite range of V . This proves the assertion at the beginning of this section. Fig. 1 d illustrates a case in which $p(V)$ is multivalued over a range in V . Such a system can exhibit both isochoric and isobaric instability, where the equilibrium curve has a *negative* slope in the $p - V$ plane.

A system with two stable phases (e.g., Fig. 1 b) may be used to illustrate the inevitability of multiple phases under certain circumstances. A characteristic feature of two-phase systems is that the cold phase cannot exist below some minimum pressure p_{\min} , while the hot phase cannot exist above some maximum pressure p_{\max} . The condition that there be two stable phases implies that $p_{\max} > p_{\min}$. Now consider a homogeneous system with a density $n_1 < \bar{n} < n_2$, as shown in Fig. 2a. Such a system is clearly unstable in its homogeneous state. However, it is always possible to stabilize the system by making it *inhomogeneous*, while keeping the mean density constant (Fig. 2b). The trick is to put most of the mass in the cold phase, with density $n_c > n_2$, while a small fraction of the matter forms a hot intercloud medium, with density $n_h < n_1$ and temperature T_h . Pressure balance

requires $n_c / n_h = T_h / T_{cl}$. If f is the filling factor in cold gas, then the mean density constraint is $\bar{n} = (1 - f)n_h + fn_c$, and f satisfies $T_{cl} / T_h \ll f \ll 1$ if $n_1 \ll \bar{n} \ll n_2$.

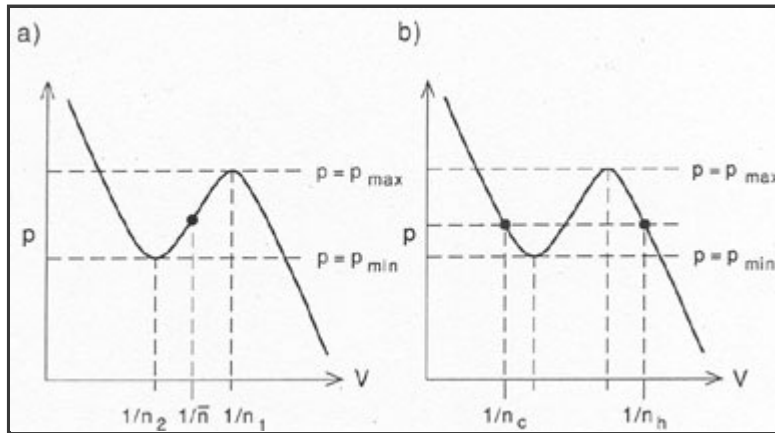


Figure 2. Inevitability of thermal instability in a system with a fixed mean density \bar{n} and variable pressure p . Equilibrium curve is identical to that in [Fig. 1b](#). a) Homogeneous state is thermally unstable. b) In the stable two-phase state, most of the mass is in the cold phase with $n_c > n_2$ and a small filling factor.

2.3. Examples of thermal phases

Field, Goldsmith and Habing (FGH: 1969) produced the first specific model for a two-phase equilibrium of the interstellar medium (ISM), in which radiative cooling is balanced by cosmic ray heating. The two phases in the FGH model include cold clouds ($T \sim 100$ K) and a warm intercloud medium ($T \sim 10^4$ K). Other heating mechanisms which may be important (probably more important than cosmic rays [Spitzer 1978]) include diffuse UV and X-ray flux, photoelectric emission by normal grains (Draine 1978; de Jong 1980; Shull and Woods 1985) or polycyclic aromatic hydrocarbons (PAHs: d'Hendecourt and Leger 1987; Lepp and Dalgarno 1988), mechanical heating (Cox 1979), magnetoacoustic waves (Spitzer 1982; Ikeuchi and Spitzer 1984), and ion-neutral friction (Scalo 1977; Ferrière, Zweibel and Shull 1988). The characteristic temperatures of the warm and cold thermal phases are insensitive to the details of the heating processes; they simply reflect the energies of the resonance and fine-structure lines, respectively, responsible for cooling the gas. Gas at $\sim 10^4$ K may exist in a range of ionization states, and McKee and Ostriker (1977) drew a distinction between the "warm neutral medium" and a "warm (photo)ionized medium" irradiated by UV from hot stars. A molecular phase at ~ 10 K is now known to contain most of the mass in the ISM of the Milky Way, but this component appears to form self-gravitating clouds which are out of pressure balance with the rest of the ISM. A phase diagram for these phases is computed by Lepp et al. (1985).

The gas which emits the broad emission lines in AGN has also been modeled as part of a stable two-phase medium (McCray 1979; Krolik, McKee, and Tarter 1981 [KMT]; Lepp et al. 1985; Krolik 1988). On the basis of observations, the line-emitting gas is inferred to be concentrated in many small clouds which fill a tiny fraction of the volume of the emission line region (Davidson 1972). Compton heating by the observed X-rays provides the minimum level of heating of the hot component of the medium; additional heating due to relativistic particles, radio frequency heating, cloud friction, and shocks may also be important (KMT). Cooling of the hot phase be homogeneous and hot or in two phases; and finally, there is usually a range of densities for which the gas *must* be in two phases (cf. [Fig. 2](#) and [Section 2.2](#)). KMT showed that unless the temperature of the hot gas in the broad line region is well above 10^8 K, most of the mass is in the hot phase, corresponding to the hot/two-phase case.

Cox and Smith (1974) pointed out that the cooling time of interstellar gas shock-heated by supernova remnants could be longer than the interval between the passage of successive shocks. This suggestion led to the three-phase model of the ISM (McKee and Ostriker 1977), in which most of the volume is occupied by shock-heated gas. This $\sim 10^6$ K gas is an example of a non-equilibrium phase, the possibility of which was foreseen by Spitzer (1956). Because it is produced dynamically, and has a temperature of order the virial temperature of the Galaxy, it has proven very difficult to determine the fate of the hot intercloud medium. It is not at all clear whether it cools radiatively in a region close to the disk (McKee and Ostriker 1977) or is vented into the halo through "chimneys" (McCray and Kafatos 1987; Norman and Ikeuchi 1989), where it undergoes a combination of adiabatic and radiative cooling (the "Galactic fountain": Shapiro and Field 1976; Cox 1981; Wang and Cowie 1988). It is also not known whether the hot gas cools sufficiently in the halo to form clouds which eventually rain down on the disk, remains hot

enough to drive a galactic wind, or somehow does both. Finally, the effects of spatial correlations among Type II supernovae (in OB associations) are just beginning to be appreciated (McCray and Kafatos 1987).

Cooling flows in elliptical galaxies and galaxy clusters are also thought to have a nonequilibrium two-phase structure. When the existence of cooling flows was first recognized (Cowie and Binney 1977; Fabian and Nulsen 1977), it was pointed out that the cooling gas should be thermally unstable to the formation of cool ($\sim 10^4$ K) filaments (Fabian and Nulsen 1977; Mathews and Bregman 1978; Cowie, Fabian and Nulsen 1980). Optical emission lines have been observed in the central regions of many cooling flows (Lynds 1970; Heckman 1981; Cowie et al. 1983; Ru, Cowie and Wang 1985; Johnstone, Fabian and Nulsen 1987; Heckman et al. 1989). However, the development of linear thermal instability is severely hampered by buoyancy (Balbus 1988; Balbus and Soker 1989), and it is not clear whether the filaments grow from finite but small perturbations or are advected inward in a highly nonlinear form (Nulsen 1986). Furthermore, the mechanism which excites the emission lines is very uncertain, and may play a role creating and maintaining the multiphase structure. Multiphase models of cooling flows have been studied by Nulsen (1986); Thomas, Fabian and Nulsen (1987); Thomas (1988); and Böhringer and Fabian (1989).

An extreme version of the cooling flow instability has been proposed to account for the masses of protogalaxies (Rees and Ostriker 1977; Silk 1977) and of globular clusters (Fall and Rees 1985). The basic idea of these models is that a self-gravitating gas cloud will fragment only when its cooling time becomes shorter than its free-fall time, and then it will develop a two-phase structure in which just enough material drops out of the hot phase to keep the cooling time roughly comparable to the free-fall time. Characteristic mass scales are determined by the Jeans mass of the cold phase in pressure balance with the hot phase. Triggering of star formation by radio lobes expanding into a protogalactic multiphase medium has been proposed (Rees 1989; Begelman and Cioffi 1989) to account for the observed radio/optical alignments in high-redshift radio galaxies (McCarthy et al. 1987; Chambers, Miley and van Breugel 1987). Cool gas in the multiphase protogalactic environment might give rise to some quasar absorption line systems (Hogan 1987) as well as the extended emission-line "fuzz" around high-redshift quasars (Rees 1988).

3. INTERACTIONS AMONG PHASES

3.1. Evaporation and condensation

Thermal conduction tries to destroy multiphase structure by erasing temperature gradients. Whether this tendency toward homogenization leads to evaporation of clouds or the condensation of hot phase onto existing clouds depends on the cooling function in the hot phase, as well as the sizes and distribution of clouds. The efficiency of thermal conduction also depends on the magnetic connectivity between the phases, which is poorly understood. Although the conductivity perpendicular to a magnetic field line is almost completely suppressed, any connection between the phases, albeit by tangled field lines, is likely to suppress the conductivity only by a factor of a few (Tribble 1989). In a fully ionized cosmic plasma the "classical" coefficient of conductivity is

$$\kappa = 5.6 \times 10^{-7} \phi_c T_e^{5/2} \text{ erg s}^{-1} \text{ K}^{-1} \text{ cm}^{-1} \quad (7)$$

(Spitzer 1962; Draine and Giuliani 1984), where the factor $\phi_c \leq 1$ allows for a reduction in the mean free path due to magnetic fields or turbulence. Equation (7) is appropriate when the electron mean free path is sufficiently short compared to $T / |\nabla T|$ that heat conduction can be treated in the diffusion approximation, $\vec{q} = -\kappa \nabla T$. When the diffusion approximation breaks down, the conductive heat flux first enters the saturated regime (Cowie and McKee 1977), $q_{\text{sat}} = 5 \phi_s c_s p$ (where $c_s = (p / \rho)^{1/2}$ is the isothermal sound speed in the intercloud medium and ϕ_s is a suppression factor similar to ϕ_c), and eventually the "suprathematic" regime (Balbus and McKee 1982), in which thermal conduction is best treated by a two-fluid approach.

The inhibition of multiphase structure by thermal conduction was first discussed by Field (1965), who found that conduction suppresses thermal instability for wavelengths shorter than a critical value which Begelman and McKee (1990) have generalized and dubbed the *Field length*,

$$\lambda_F \equiv \left(\frac{\kappa T}{n^2 \mathcal{L}_M} \right)^{1/2}, \quad (8)$$

where $\mathcal{L}_M \equiv \text{Max}(\Lambda, \Gamma / n)$. λ_F is the maximum length scale across which thermal conduction can dominate over radiative heating and cooling. Therefore, the thickness of a conductive interface with a radius of curvature r_c is $\sim \min(\lambda_F, r_c)$ (McKee and Cowie 1977). This implies that conduction into clouds with radii smaller than λ_F is unaffected by heating and cooling processes

in the surrounding medium. Such "small" clouds always evaporate (Graham and Langer 1973; Cowie and McKee 1977), at a rate given by

$$\dot{M}_{ev} = \frac{16\pi}{25} r_c \frac{\kappa T}{c_s^2} \quad (9)$$

in the classical conduction limit. Clouds with radii larger than λ_F have conductive interfaces whose structures are independent of the cloud size; such interfaces are dominated by the balance between conduction and heating/cooling, and may be treated as plane-parallel.

Steady plane-parallel conduction fronts have been analyzed by Zel'dovich and Pikel'ner (1969), Penston and Brown (1970), and McKee and Begelman (1990). Ballet, Arnaud and Rothenflug (1986) and Böhringer and Hartquist (1987) studied non-equilibrium ionization in steady evaporative flows. Time-dependent mass exchange has been analyzed in one dimension by Doroshkevich and Zel'dovich (1981), by Balbus (1986b), who included magnetic fields, and by Borkowski, Balbus and Frstrom (1989) who also studied the ionization structure. If the hot phase is cooling (and is thermally unstable) then a cooling wave of fixed thickness propagates into the hot gas following an evaporative transient. Doroshkevich and Zel'dovich (and Böhringer and Fabian 1989) used this result to argue that steady-state evaporation solutions are incorrect, i.e., that all clouds embedded in a cooling background medium should condense, not evaporate. However, the evaporative transient lasts until the temperature gradient relaxes to the Field length, and the timescale for this to occur is the cooling time. The evaporative solutions found by Cowie and McKee (1977) persist over a time scale which is short compared to the cooling time, but long compared to the time required to set up the evaporation flow. If the hot phase is thermally stable, then there exists a "saturated vapor pressure" p_{sat} above which "large" clouds condense, and below which they evaporate (Penston and Brown 1970). Zel'dovich and Pikel'ner (1969) devised an approximate method for calculating the evaporation rate when $p \neq p_{sat}$, which was refined and generalized to spherical clouds by McKee and Begelman (1990).

3.2. Ablation

The motion of clouds with respect to the ambient hot medium leads to Kelvin-Helmholtz and Rayleigh-Taylor instabilities, which can break up the clouds into smaller pieces and accelerate mass exchange between the phases. Both instabilities operate on time scales $t_i \sim (\rho_c / \rho_h)^{1/2} r_c / v$, where v is the relative speed between the cloud and the hot medium and $\rho_c / \rho_h \sim T_h / T_c$ in pressure equilibrium. Most studies have concentrated on the fate of a cloud overtaken by a strong supernova or spiral density-wave shock (Woodward 1976; Nittman, Falle and Gaskell 1982; Heathcote and Brand 1983; McKee 1988; Klein, McKee and Colella 1989). In this case t_i is of the same order as the "cloud-crushing" time, t_{cc} , which is the time scale required for a secondary shock to be driven into a cloud once it is overrun by the main shock (McKee 1988). The cloud destruction process is accelerated by the significant pressure differential between the sides of the cloud and its front and back (Nittman, Falle and Gaskell 1982). The unbalanced forces cause the cloud to "pancake", i.e., to spread sideways, and the increase in cross-section speeds up the momentum deposition which tears apart the cloud. Pressure fluctuations and vorticity generation arising from the interactions of multiple shocks also play an important role in cloud disruption (Klein, McKee and Colella 1989).

The time scale for ablated cloud material to be effectively mixed with the intercloud medium should lie somewhere between t_i and the hydrodynamic drag time, $t_d \sim (\rho_c / \rho_h) r_c / v$. Nulsen (1982), using the longer time scale t_d , estimated that cold gas would be ablated from a cloud at a rate $\dot{M}_{ab} \sim \pi r_c^2 \rho_h v$. If thermal conduction were negligible, the cloud would leave behind a cylindrical "trail" with a radius $\sim r_c$, containing cold material with a mean density $\langle \rho \rangle_{tr} \sim \rho_h$. If the ablated gas is well-mixed with the hot phase downstream of the cloud, as we might expect from a turbulent ablation process, then the global time scale for cooling the hot phase by ablation is simply the time required for the trails to fill space, $t_{ab} \sim r_c / \pi f v$, where f is the filling factor in clouds. t_{ab} is shorter than the cloud disruption time if the clouds contain more mass than the hot phase, and it is longer than the saturated evaporation time by a factor $\sim \mathcal{M}^{-1}$, where \mathcal{M} is the Mach number of cloud motion relative to the hot phase.

For diffuse interstellar clouds moving through the hot phase of the ISM in the Milky Way, $\mathcal{M} \sim 0.1$. According to the Nulsen (1982) model, ablation from subsonically moving clouds is a less important mechanism for destroying clouds than conduction in the saturated limit, but may be more important than conduction in the classical limit, i.e., for large clouds. For clouds moving nearly sonically, e.g., randomly moving clouds in the spheroidal component of a galaxy, hydrodynamical instabilities are probably the most efficient mechanism for shredding clouds to the point where thermal mixing via conduction is very efficient.

Lateral expansion of the cloud can shorten the hydrodynamic drag time considerably (Nittman, Falle and Gaskell 1982; Klein,

McKee and Colella 1989). Klein, McKee and Colella find that the drag time is of order t_i for density contrasts ρ_c / ρ_h as high as 100, but for much larger density contrasts the cloud is torn apart before it slows significantly. These calculations suggest that mixing can occur much more rapidly than predicted by the Nulsen (1982) model. Further numerical simulations capable of following the mixing process with high resolution are clearly needed to test the basic assumptions of any ablation model.

4. CONSEQUENCES OF MASS EXCHANGE

4.1. Mass exchange vs. radiative heating/cooling

When the bulk of the energy content and the bulk of the mass content reside in different phases, a relatively small amount of mass or energy transfer between phases can have a large effect on the structure of a multiphase medium. Such a situation is believed to exist in the three-phase model of the ISM (McKee and Ostriker 1977), where the hot phase occupies most of the volume while most of the mass is in cold clouds. Physically, the effect of mass exchange (either by conduction or ablation followed by effective mixing) is to cool the hot phase, since a fixed amount of energy is being distributed among a larger number of particles. Since radiative heating and cooling depend on both density and temperature, mass exchange can affect the radiative evolution of the medium as well. We can illustrate the global consequences of mass and energy exchange between phases by considering a medium with uniform pressure and subsonic motions, in which mass exchange is driven by thermal conduction. The approximate time-dependent equations governing the medium are then

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 \quad (10)$$

$$\frac{3}{2} \frac{\partial p}{\partial t} + \frac{5}{2} p \nabla \cdot \vec{v} = (n\Gamma - n^2\Lambda) - \nabla \cdot \vec{q}, \quad (11)$$

where \vec{q} represents the conductive heat flux. Note that these equations are appropriate in the single-fluid limit, corresponding to classical or mildly saturated conduction (Cowie and McKee 1977): in the highly saturated suprathermal limit (Balbus and McKee 1982), both the single-fluid approximation and the assumption of pressure balance break down. The evolution of the medium is driven by the terms on the right-hand-side of eq. (11), and may be dominated either by the effects of conduction or by the effects of radiative heating and cooling.

There is a crucial distinction between the heat flux term and the radiative loss term in the energy equation: the energy entering or leaving a volume of radius r due to conduction is proportional to $r^2 q$ ($\propto r$ for classical conduction) whereas that due to heating or cooling is proportional to r^3 . Thus, there is a critical length scale which enters the problem, which turns out to be the Field length, λ_F (eq. [8]), when mass exchange is driven by classical conduction.

The temperature structure of the intercloud medium in a system of clouds extending over a region of size R depends on the ratio of the Field length to R . Balbus (1985) developed an elegant electrostatic analogy for an ensemble of clouds in a hot intercloud medium under conditions in which radiative heating and cooling are negligible and the temperature is specified on the boundary of the ensemble. This corresponds to the case $R \ll \lambda_F$. In a steady state, the evaporation rate and the temperature structure in the intercloud medium are then determined by a solution of Laplace's equation with Dirichlet boundary conditions. In the complementary case, $R \gg \lambda_F$, global heat flows are insignificant and the temperature structure of the intercloud gas is determined by a competition between cloud evaporation on the one hand and heating and cooling on the other. Numerically, the Field length is

$$\lambda_F = 2.4 \left(\frac{\phi_c^{1/2} T_6^{7/4}}{n \mathcal{L}_{M-23}^{1/2}} \right) \text{ pc}, \quad (12)$$

where $T_6 \equiv T/10^6$ K and $\mathcal{L}_{M-23} \equiv \mathcal{L}_M / (10^{-23} \text{ erg cm}^3 \text{ s}^{-1})$ is normalized to a characteristic value of the radiative cooling rate for astrophysical plasmas.

To quantify the competition between mass exchange and radiative heating/cooling, consider the spatially averaged effect of cloud evaporation on the hot phase. This is meaningful only when the characteristic temperature of the hot phase changes over a length scale which exceeds the mean intercloud separation r_0 . This condition is guaranteed to be satisfied when r_0 is smaller than λ_F . When r_0 exceeds λ_F , then mass exchange cannot compete with radiative heating and cooling anyway, so the point is moot. If there are $n_{cl} (\equiv 3/4\pi r_0^3)$ clouds per unit volume evaporating at a mean rate \dot{M}_{ev} per cloud, then the density of the intercloud

medium changes at a rate

$$\dot{n}_{\text{ev}} = \mathcal{N}_{\text{cl}} \frac{\dot{M}_{\text{ev}}}{\mu_H}, \quad (13)$$

where \dot{M}_{ev} is negative for condensation. We may then use \dot{n}_{ev} to define an *effective evaporative cooling rate*,

$$n_{\text{h}}^2 \Lambda_{\text{ev}} \equiv \frac{5}{2} p \left(\frac{\dot{n}_{\text{ev}}}{n_{\text{h}}} \right) \quad (14)$$

(Begelman and McKee 1990). The evaporative cooling coefficient Λ_{ev} is analogous to the radiative cooling coefficient Λ in that both reduce the specific entropy s , but there are crucial differences between the two: Λ reduces the total entropy of a given volume of intercloud gas, whereas Λ_{ev} increases it; Λ reduces the energy density of the intercloud gas, whereas Λ_{ev} leaves it unchanged. Because of these distinctions, Λ_{ev} should not be included in the net radiative cooling function \mathcal{L} . However, the relative impact of mass exchange and radiative heating/cooling on the thermal state of the intercloud medium can be expressed by the *radiation/evaporation ratio* (Begelman and McKee 1990),

$$\mathcal{R} \equiv (\mathcal{L}/\Lambda_{\text{ev}}). \quad (15)$$

When $\mathcal{R}_M \equiv \mathcal{L}_M / \Lambda_{\text{ev}}$ is $\gg 1$ ($\ll 1$), then radiative heating and cooling (mass exchange) determines the thermal state of the intercloud medium.

One can express \mathcal{R} in terms of quantities which characterize the structure of the two-phase medium. Writing the evaporation rate in the form $\dot{M}_{\text{ev}} = 4\pi r_c^2 \rho_{\text{h}} c_s f$, where c_s is the isothermal sound speed in the hot phase, we have

$$\mathcal{R} = \frac{2}{15F} \frac{r_0^3 n_{\text{h}}^2 \mathcal{L}}{r_c^2 p c_s}. \quad (16)$$

r_0 may be eliminated in favor of the cloud filling factor f (assumed to be $\ll 1$) by substituting r_c/f for r_0^3 / r_c^2 . In the limit of saturated evaporation $F \sim$ a few (Cowie and McKee 1977), while $F \sim \mathcal{M} / 4$ in the Nulsen (1982) model of ablation. In the classical conduction limit, F is twice the "saturation parameter" σ'_0 derived by Cowie and McKee (1977), with the result that

$$\mathcal{R}_M = \frac{5}{6} \left(\frac{r_0^3}{r_c \lambda_F^2} \right). \quad (17)$$

Cloud evaporation thus determines the intercloud temperature for $r_c > r_0^3 / \lambda_F^2$. The corresponding condition on the filling factor is

$$f > (r_c / \lambda_F)^2 \quad (18)$$

or $f > (r_0 / \lambda_F)^6$; for $f \sim 0.03$, as in the ISM, this will be true if $\lambda_F \gtrsim 2r_0$. In terms of the sound-crossing time across r_c (measured in the hot phase), $t_s \sim r_c / 2c_s$, and the radiative cooling time in the hot phase, $t_c \sim 5kT_{\text{h}} / 2n_{\text{h}} \Lambda(T_{\text{h}})$, we may express the condition for mass exchange to dominate globally in the form

$$f \gtrsim \frac{t_s}{F t_c}. \quad (19)$$

Writing (18) in the form $r_0 < r^{1/3} c \lambda_F^{2/3}$, we see that the intercloud spacing in a conduction-dominated medium must also be smaller than the Field length. This has important consequences for the thermal stability of the hot phase in a

conduction-dominated system. Since λ_F is roughly the minimum wavelength which is thermally unstable (Field 1965) potentially unstable regions must contain many clouds. The conduction-modified condition for thermal instability is obtained simply by including Λ_{ev} in Balbus's (1986a) criterion (eq. [4]):

$$\left\{ \frac{\partial}{\partial T_h} \left[\frac{n_h (\Lambda + \Lambda_{ev}) - \Gamma}{T_h} \right] \right\}_A < 0 \quad (20)$$

(Begelman and McKee 1990). $n_h \Lambda_{ev} / T_h$ is generally an increasing function of T_h ; for isobaric perturbations $n_h \Lambda_{ev} / T_h \sim q \propto T_h^{-7/2} (T_h^{1/2})$ for classical (saturated) conduction. Therefore, evaporation has a *stabilizing* influence on the hot phase, and in a conduction-dominated medium thermal instability will be inhibited by the presence of evaporating clouds (Begelman and McKee 1990). This will be true even if the radiative processes place the hot phase in a thermally unstable regime. However, it should be noted that the cooling time scale in the hot phase of a conduction-dominated medium is shorter than the radiative cooling time scale. Therefore, conduction cannot stabilize a hot phase over a time scale which is longer than the time scale for radiative thermal instability. However, it can lead to the hot phase cooling down somewhat before the onset of thermal instability. Since thermal conduction generally becomes less important at low temperatures, such a system may evolve to a state in which evaporative cooling no longer dominates, whereupon thermal instability may occur. Since the Field length is generally a strongly increasing function of temperature, the operation of thermal instability may lead to the production of smaller and more closely spaced clouds than would have formed in the hotter medium.

It is instructive to apply the ideas discussed above to the three-phase model of the ISM (McKee and Ostriker 1977). The three-phase ISM consists of cold HI clouds surrounded by warm HI and HII envelopes, all embedded in a pervasive hot ionized medium (HIM). The physical conditions in the HIM are governed by mass exchange with the clouds and energy injection by supernovae. The model is intrinsically time-dependent: A given element of gas is compressed and heated by SNRs at intervals of about 5×10^5 yr, and this makes its evolution difficult to analyze using concepts developed to treat steady-state or slowly evolving systems. Nonetheless, it is of interest to evaluate the Field length and the radiation/evaporation ratio. Using the fit to the cooling function of Raymond, Cox and Smith (1976) for cosmic abundances, $\Lambda(T) \approx 1.6 \times 10^{-19} T^{-1/2}$ erg cm³ s⁻¹ (10^5 K < T < 4×10^7 K), multiplied by an enhancement factor $\beta \equiv 10 \beta_1 \approx 10$ to take account of nonequilibrium ionization and density inhomogeneity near conduction fronts (McKee and Ostriker 1977), we have $\lambda_F = 44 (\phi_c^{1/2} T_6^3 / \tilde{p}_4 \beta_1^{1/2})$ pc, where $\tilde{p}_4 \equiv p / 10^4 k$ and $T_6 \equiv T_h / 10^6$ K. We can write condition (18) in the form

$$f > 5 \times 10^{-3} \frac{\beta_1}{\phi_c} \left(\frac{r_c}{1 \text{ pc}} \right)^2 \frac{\tilde{p}_4^2}{T_6^6}. \quad (21)$$

It is evident that the relative importance of conductive and radiative energy exchange is sensitive to conditions in the hot phase, particularly to T_h , as well as to the filling factor and typical size of clouds. Under the conditions deduced by McKee and Ostriker ($T_6 = 0.45$, $\tilde{p}_4 = 0.36$, $\beta_1 = 1$, $r_c = 2.1$ pc, and $f = 0.23$), condition (21) is marginally satisfied. Equivalently, the radiation/evaporation ratio is given by $R = 0.38$, which implies that evaporative cooling dominates radiative cooling and that the HIM is thermally stable. (The argument that evaporation can stabilize the HIM is originally due to McCray [1986].) However, even a small amount of cloud ablation by hydrodynamic processes (Section 3.2) or a slight increase in the HIM temperature would lead to a drastic increase in the energetic importance of evaporation. If the typical clouds were sufficiently small that conduction were saturated, i.e., for

$$r_c < 0.5 \phi_c \frac{T_6^3}{\tilde{p}_4} \text{ pc}, \quad (22)$$

(Cowie and McKee 1977; Balbus and McKee 1982), the appropriate version of condition (19) would be

$$f > 7 \times 10^{-3} \frac{\beta_1}{\phi_c} \left(\frac{r_c}{1 \text{ pc}} \right) \frac{\tilde{p}_4}{T_6^3}. \quad (23)$$

In the McKee-Ostriker picture, each region of HIM is overrun by another supernova remnant before it has time relax to a stationary state with $\mathcal{R} \gtrsim 1$. Stochastic heating by SNR shocks thus leads to discontinuous trajectories in the $p - V$ plane, and

heating is balanced by radiative cooling. In the galactic fountain model (Shapiro and Field 1976; Wang and Cowie 1988), the heat is advected into the galactic halo. If clouds are effectively ablated then the accelerated lowering of T_h by evaporation may allow the large tracts of the ISM to cool to a homogeneous state at $T \lesssim 10^4$ K. The intercloud medium remains thermally stable until radiative cooling begins to dominate over evaporative cooling, at $T_h \lesssim 10^5$ K. Since the Field length is a strongly increasing function of T_h , the onset of instability at a reduced temperature may lead to the formation of sub-parsec size clouds.

4.2. Evolution of multiphase systems

By averaging the equations of mass and energy conservation over a volume (V) which contains many clouds, one can derive equations for the global evolution of the hot intercloud medium in the presence of mass exchange with embedded clouds (Begelman and McKee 1990). The mass of intercloud gas in V is $\bar{n}_h \mu_H V$, where \bar{n}_h is the mean density of the intercloud gas in V . Choosing the volume V to comove with the intercloud gas implies that this mass can change only by cloud evaporation, at a rate $\dot{n}_{ev} \mu_H V$. Mass conservation for the intercloud gas then becomes

$$\frac{d\bar{n}_h}{dt} + \bar{n}_h \left(\frac{\dot{V}}{V} \right) = \dot{n}_{ev}. \quad (24)$$

Integration of equation (11) over V then implies

$$\frac{3}{2} V \frac{dp}{dt} + \frac{5}{2} p \dot{V} = - \int_V n^2 \mathcal{L} dV - \int_S \bar{q} \cdot d\vec{S}, \quad (25)$$

where s is the surface bounding V . By assuming that the characteristic dimension of the averaging volume is large compared to the Field length, we ensure that the conductive heat flux term in equation (25) is negligible compared to the heating and cooling term. The global energy equation then simplifies to

$$\frac{dp}{dt} = - \frac{2}{3} \langle n^2 \mathcal{L} \rangle_V - \frac{5}{3} p \left(\frac{\dot{V}}{V} \right), \quad (26)$$

where $\langle \rangle_V$ denotes an average over the volume V . Note that mass exchange does not enter this equation: mass exchange alters the density and temperature of the intercloud gas, but not its pressure.

Additional constraints are required to solve for the evolution of a specific system. Since the cloud evaporation rate depends on the typical cloud size, it is necessary to have an equation for the evolution of r_c in time. There are also likely to be externally imposed constraints on the intercloud medium; Begelman and McKee (1990) considered two limiting cases. In the *isochoric* limit, the comoving volume V is held constant as the system evolves. If one assumes that the clouds are fixed as well, then the mean density \bar{n} also remains constant. This condition would apply in a system in which the sound crossing time R/c_s is long compared to both the characteristic heating/cooling time and the evaporation time. Such a situation might apply locally within a supersonic accretion flow or wind. Setting $\dot{V} = 0$ in eqs. (24) and (26), we obtain

$$\frac{dp}{dt} = - \frac{2}{3} n_h^2 \mathcal{L}, \quad (27)$$

and

$$\frac{dn_h}{dt} = \dot{n}_{ev}. \quad (28)$$

In the *isobaric* limit, the intercloud medium can exchange mass with a reservoir in order to maintain a constant pressure, so we set $p(t) = \text{constant}$. Such a situation might apply, for example, if the system were in contact with an X-ray heated wind above an accretion disk (Begelman, McKee and Shields 1983). We then have

$$\frac{dn_h}{dt} = \dot{n}_{ev} + \frac{2}{5} \left(\frac{n_h}{p} \right) n_i^2 \mathcal{L}. \quad (29)$$

The instantaneous state of the intercloud medium can be described by the location of a point in the $p - V$ plane (cf. Fig. 1); $V \equiv n_h^{-1}$ is the specific volume of the intercloud gas. The radiative equilibrium curve $n_h^2 \mathcal{L} = 0$ divides the plane into two regions: above the curve, radiative cooling exceeds the external heating, whereas below the curve the converse is true. The net cooling rate $n_h^2 \mathcal{L}$ may be assumed to be a known function of p and V everywhere on the plane. However, the evaporation rate \dot{n}_{ev} also depends on the distribution of cloud sizes and separations.

The character of a trajectory in the $p - V$ plane is determined by the relative importance of energy exchange and mass exchange, which is expressed quantitatively by the radiation/evaporation ratio \mathcal{R} (eq. [16]). in the isochoric case, the slope of trajectories in the $p - V$ plane is governed by the ratio of equations (27) and (28):

$$\frac{d \ln p}{d \ln V} = \frac{5}{3} \mathcal{R}(p, V). \quad (30)$$

Trajectories are nearly vertical if radiative cooling and heating are dominant ($|\mathcal{R}| \gg 1$), and nearly horizontal if mass exchange is dominant ($|\mathcal{R}| \ll 1$). It is immediately obvious that trajectories must be locally horizontal ($dp / dV = 0$) where they cross the equilibrium curve ($\mathcal{L} = 0$), provided that $\dot{n}_{ev} \neq 0$ at the point of crossing. Points at which $\mathcal{L} = \dot{n}_{ev} = 0$ represent stationary states. The temperature evolves as

$$\frac{d \ln T}{dt} = -\frac{2}{5} \frac{n_h^2 \mathcal{L}}{p} \left(\frac{5}{3} + \frac{1}{\mathcal{R}} \right). \quad (31)$$

Trajectories with $\mathcal{R} = -3/5$ are isothermal. Isobaric trajectories are constrained to be horizontal in the $p - V$ plane. The direction (and rate) of motion is given by equation (29), which may be written in the form

$$\frac{d \ln V}{dt} = -\frac{2}{5} \frac{n_h^2 \mathcal{L}}{p} \left(1 + \frac{1}{\mathcal{R}} \right). \quad (32)$$

Since $T \propto pV \propto V$ in the isobaric case, this equation also describes the temperature evolution of the system. The point $\mathcal{R} = -1$ represents a steady state for the hot phase, although mass continues to be lost (if $\mathcal{L} < 0$) or gained (if $\mathcal{L} > 0$) by clouds in this state (unlike the "true" steady state $\mathcal{L} = \dot{n}_{ev} = 0$ in the isochoric case). For a system dominated by radiative heating or cooling ($|\mathcal{R}| \gg 1$), evolution is leftward above the thermal equilibrium curve and rightward below the curve. In a system dominated by mass exchange ($|\mathcal{R}| \ll 1$), evolution is leftward for evaporation and rightward for condensation. A more detailed discussion of $p - V$ plane trajectories may be found in Begelman and McKee (1990), who considered the specific case of a gas heated by Compton scattering and cooled by bremsstrahlung and the inverse Compton effect. They also discuss the stability properties of evolving systems.

5. CONCLUSIONS

Multiple thermal phases are known to exist in many astrophysical systems. The reasons for their existence are understood in general terms, but the detailed properties of specific multiphase systems are poorly known. We can look forward to the further development of multiphase models for the ISM in elliptical galaxies, cooling flows in clusters of galaxies, the intergalactic medium, and protogalactic environments. The structures of multiphase media are sensitive to the rate of mass exchange between phases, which tends to lower the temperature of the hot phase and render it thermally stable. Unfortunately, mass exchange through thermal conduction depends on the topology of conduction fronts and on the magnetic connectivity of the phases, neither of which is understood. However, mixing of the phases may be driven by hydrodynamic instabilities at a rate much faster than that due to thermal conduction. Rapidly improving hydrodynamic codes with a high dynamic range in spatial resolution (e.g., using an adaptive mesh) should clarify some of the physics of the ablation process within the next few years.

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