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CHAPTER 15
Clusters of Galaxies

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1. INTRODUCTION

OUR KNOWLEDGE of the large-scale structure of the Universe is limited by our ability to recognize and observe remote objects, and to determine their distances and physical properties. Clusters of galaxies may be the most fundamental condensations of matter in space. In any case, they are the systems which we can most conveniently recognize, and for which estimates of at least relative distances are possible. The study of clusters of galaxies, therefore, in addition to its own intrinsic interest, may well provide the clues to our understanding of the Universe as a whole.

2. NUMBERS AND CATALOGS OF CLUSTERS

Long before the era of extragalactic astronomy it was recognized that the small-scale distribution of "nebulae" in space is not random. Even the small sample of the 35 Messier objects now recognized as galaxies exhibits this nonrandomness; nearly half of these objects are in the vicinity of the Virgo cluster. Plots of the distribution on the sky of "nebulae" listed in John Herschel's *General Catalogue*, or in Dreyer's *New General Catalogue* and *Index Catalogue*, reveal several pronounced concentrations now recognized as nearby clusters of galaxies. Two of these, the Perseus cluster and the Coma cluster, were described in some detail by Wolf (1902, 1906) early in the century. Also, Wirtz (1923, 1924a, b), who supplemented the distribution of NGC and IC objects with data from the surveys of Fath, Curtis, and himself, called attention to several conspicuous well-defined centers of clustering.

Following Hubble's demonstration of the nature of galaxies in 1924, several systematic surveys of the distribution of galaxies were made. Among these are the extensive surveys by Shapley and his co-workers at Harvard in the two decades following 1930. At least four distinct clusters - the Virgo cluster, the Ursa Major cloud, and two clusters in Fornax - are apparent in the distribution of the bright galaxies in the Shapley-Ames catalog (1932). Shapley himself (1933) catalogued and described 25 individual clusters. The Harvard surveys to faint limiting magnitudes in the southern hemisphere with the Bruce telescope revealed still greater clustering, and Shapley (e.g., 1957) has called attention to a major unevenness in surface distribution, which apparently cannot be described as local clustering but which suggests "metagalactic structure" or superclustering.

Hubble's (1934) fundamental investigation of the distribution of galaxies over the sky provided further information on the clustering tendency. The survey was conducted on 1283 small selected regions of the sky which were intentionally chosen to avoid most of the well-known clusters; nevertheless, from the scanty evidence available to him, Hubble estimated there to be one great cluster (with hundreds of member galaxies) for every 50 square degrees of the sky. He estimated that if the great clusters averaged 500 members each, they would account for about 1 percent of the total number of observed galaxies. Even after omitting the great clusters from his investigation, however, Hubble found the galaxian distribution to be nonrandom; he interpreted a skewness in the frequency distribution of the numbers of fields with various numbers of counted galaxies as a tendency toward small-scale clustering (see Hubble 1936a). According to Hubble, the groups and clusters could not be merely superposed on a random distribution of isolated galaxies; either condensations in the general field produced the clusters, or evaporation of galaxies from clusters populated the general field.

Meanwhile, additional individual clusters were discovered, often by accident. Tombaugh (1937) described three at low galactic latitudes. Zwicky discovered several new clusters with the 18-inch Schmidt telescope on Palomar Mountain, and investigated these and other clusters with that instrument, and later with the 48-inch Schmidt and 200-inch telescopes (for the best summary, see Zwicky 1957). The general prevalence of clustering led him (Zwicky 1938) to propose that all galaxies belong to clusters and that the Universe can be regarded as divided into cluster "cells" with a mean diameter of about 7.5×10^6 pc. (Zwicky's estimate was based on the old distance scale; for the Hubble constant H = 50 km s^{-1}, the diameter of a "cluster cell" would be about 7.5×10^7 pc.)

Since World War II, two photographic surveys of the sky have demonstrated conclusively that clusters of galaxies are extremely numerous - far more so than most investigators had believed; these are the Lick 20-inch *Astrographic Survey* and the National Geographic Society-Palomar Observatory *Sky Survey*. From counts of galaxy images on the Lick photographs, Shane and his collaborators (Shane and Wirtanen 1954; Shane 1956a; Shane, Wirtanen, and Steinlin 1959; Shane and Wirtanen 1967) have prepared catalogs and charts of the surface density of galaxies (per square degree). They have called attention to many striking clusters and clouds of galaxies, and even to apparent superclusters. The Lick counts have been analyzed statistically under the direction of Scott and Neyman at the Berkeley Statistical Laboratory (Neyman and Scott 1952; Neyman, Scott, and Shane 1953, 1954; Scott, Shane, and Swanson 1954). It was found that the serial correlation between counts in $1° \times 1°$ squares persists to square separations of about 4°, and that it has almost the same value in both galactic polar caps. From the shape of the quasi-correlation function, the statisticians derived parameters that describe the scale and amplitude of the clustering. They then applied these parameters to the manufacture of a "synthetic" field of galaxy images based on a model that assumes that all galaxies are in clusters. Comparison of the synthetic field with fields plotted from actual plates showed a striking similarity between the prediction of the model and the observed distribution of galaxies; if anything, the actual fields displayed a slightly greater clustering tendency than the synthetic one. Neyman, Scott, et al. found typical clusters to have populations of the order 10^2 and diameters of a few million parsecs.

Limber (1953, 1954) also investigated the distribution of galaxies from Shane and Wirtanen's counts on the Lick plates and fitted it to a model in which the spatial density of galaxies is a smoothly varying function of position in space. Limber thereby derived an independent estimate of the scale of clustering. Although his procedure was criticized by Neyman and Scott (1955) on the grounds that the discrete nature of galaxies is incompatible with a smoothly varying function describing their distribution among volume cells in space, Limber's results are in qualitative agreement with those of Neyman, Scott, et al. A model which takes account of the discrete fluctuations that arise because of the discrete character of the distribution of galaxies as well as those that arise from clustering, and which also applies to a distribution that exhibits clustering but that does not consist entirely of discrete and independent clusters, has been suggested by Layzer (1956).

Tens of thousands of discrete groups and clusters of galaxies are easily identified on the Palomar *Sky Survey* photographs. The writer (Abell 1958) has catalogued 2712 of the very richest of these, and has analyzed the distribution of 1682 clusters that comprise a more or less homogeneous sample chosen from the catalog. Also from the Palomar plates, Zwicky and his associates have prepared a far more extensive *Catalogue of Galaxies and Clusters of Galaxies* (Zwicky, Herzog, Wild, Karpowicz, and Kowal 1961-68). Some statistics on the sizes of the largest of these clusters and on the area of the sky coveted by them have been published (Zwicky and Rudnicki 1963, 1966; Zwicky and Berger 1965; Zwicky and Karpowicz 1965, 1966). Finally, Klemola (1969) and Snow (1970) list an additional 78 groups and clusters of southern-hemisphere galaxies discovered on plates taken for the Yale-Columbia proper-motion survey.

Thus it is recognized today that clustering of galaxies is a dominant tendency, and may be fundamental. It is, in fact, not impossible that all or nearly all galaxies belong to clusters, or at least that they were originally formed in clusters. We speak of the general "field" or "background" of noncluster galaxies, but the extent to which such a "field" has physical significance is not known at present. As was shown by the Berkeley statistical investigation of the Lick counts, the impression of a field of noncluster objects can be created by many clusters and groups (in many of which only the one or two brightest members may be visible) seen overlapping in projection. Only those systems that stand out conspicuously against the field (whatever its nature), either because they are unusually rich aggregates or because their members lie in very close proximity to each other in comparison to intergalactic distances, will be recognized as discrete groups or clusters. Obviously small groups will not be identified unless they are relatively nearby. Even rich clusters are increasingly difficult to distinguish from the field at increasing distances. Our knowledge about clusters, therefore, tends to be biased toward the richest systems, and even for them it is never possible to say with certainty which galaxies (except statistically) are cluster members.

The very definition of a cluster or group of galaxies, therefore, is not a trivial matter. Such properties as total sizes, total populations, distributions of member galaxies, and luminosity functions depend very critically on how the clusters are defined and how their members are distinguished from the field. Several investigators (e.g., Zwicky and Abell) have given rather specific operational definitions of clusters for the purposes of their respective studies. Many of the differences in opinion among them concerning the nature of clusters stem from the fact that they have defined clusters in different ways. It is beyond the scope of this review to attempt to provide a definitive definition of a cluster of galaxies. We shall, however, try to be specific about the definitions and assumptions on which the results discussed depend.

3. OBSERVED PROPERTIES OF CLUSTERS

3.1. Types of Clusters

Clusters of galaxies display a wide variety of morphological forms, ranging from rich aggregates of thousands of members to the relatively poor groups, like the Local Group which contains only 17 to 20 known members, or even to double or triple systems if these can be classed as clusters. The smaller groups appear to be by far the most numerous, but at present there exist no reliable data on the actual relative numbers of rich clusters and poor groups. Enough is known, however, to classify the clusters into certain broad categories. For the purposes of the *Catalogue of Galaxies and Clusters of Galaxies*, Zwicky (Zwicky, Herzog, Wild, Karpowicz, and Kowal 1961-1968) classifies clusters as *compact*, *medium compact*, and *open*. He defines a compact cluster as one with a single pronounced concentration of galaxies, in which 10 or more galaxies appear (in projection) to be in contact; a *medium compact* cluster is one with a single concentration within which galaxies appear to be separated by several of their diameters, or in which there are several pronounced concentrations of galaxies; an *open* cluster is one without any pronounced peak of population, but which appears as a loose cloud of galaxies superposed on the general field.

A more recent classification of galaxian clusters was suggested by Morgan (1961) on the basis of his study of the 20 nearest clusters in Abell's (1958) catalog. He found that the clusters investigated could be divided into two categories, according to the types of galaxies encountered among their brightest members: (i) those containing appreciable numbers of galaxies of minor central concentration of light (late spiral and irregular galaxies); and (ii) those containing few or none of the latter.

The classifications assigned to clusters by the Morgan and Zwicky systems are not independent, but are strongly correlated. The main features of both systems are preserved by simply classifying clusters as *regular* or *irregular*. Although the demarcation between the two classes is not a sharp one, they do uniquely describe many of the morphological characteristics of most clusters. *Regular* clusters are all rich, having populations of the order of 10^3 or more in the interval of the brightest 6 magnitudes. They show high central concentration and marked spherical symmetry. Their memberships consist entirely, or almost entirely, of galaxies without conspicuous dust - E and S0 galaxies. Examples are the famous clusters in Coma and Corona Borealis (fig. 1) (Abell catalog numbers 1656 and 2065, respectively). Most of Zwicky's compact clusters and Morgan's type ii clusters belong to this class. *Irregular* clusters range from poor groups, like the Local Group, to relatively rich aggregates like the Virgo cluster or the Hercules cluster (fig. 2) (Abell number 2151). They contain little or no spherical symmetry, and no marked central concentration, although multiple condensations are often present. These clusters usually contain galaxies of all types, including appreciable numbers of late-type spirals and irregulars. To this class belong Zwicky's medium-compact and open clusters, and Morgan's type i clusters. The principal features of regular and irregular clusters are summarized in table 1 ; explanatory discussion follows.

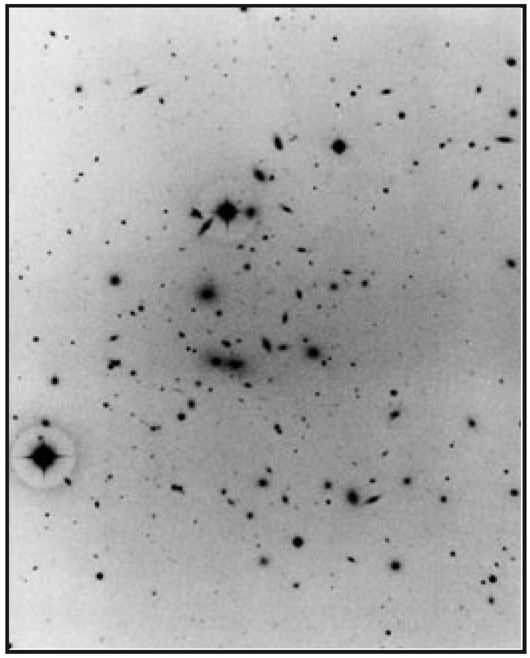

Figure 1. The regular cluster, number 2065 (Corona Borealis), photographed with the 200-inch telescope. Scale: 1 mm = 3.9".

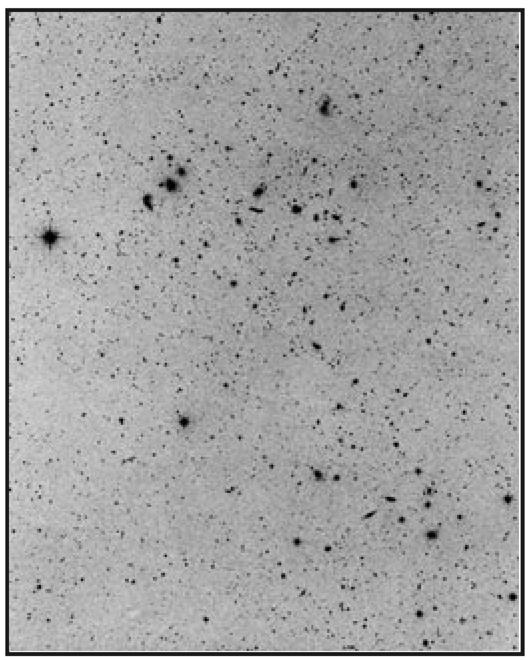

Figure 2. The irregular cluster, number 2151 (Hercules), photographed with the 48-inch Schmidt telescope. Scale: 1 mm = 22.3".

Table 1. Typical characteristics of regular and irregular clusters

Parameter	Regular Clusters	Irregular Clusters
Symmetry	Marked spherical symmetry	Little or no symmetry
Concentration	High concentration of members toward cluster center	No marked concentration to a unique cluster center; often two or more nuclei of concentration are present
Types of galaxies	All or nearly all galaxies in the first 3 or 4 magnitude intervals are elliptical and/or S0 galaxies.	All types of galaxies are usually present except in the poor groups, which which may not contain giant ellipticals. Late-type spirals and/or irregular galaxies present
Number of member galaxies in range of brightest 7 mag	Order of 10^3 or more	Order of 10^1 to 10^3
Diameter (Mpc)	Order of 1-10	Order of 1-10
Presence of subclustering	Probably absent or unimportant	Often present. Double and multiple systems of galaxies common
Dispersion of radial velocities of members about mean for cluster	Order of 10^3 km s^{-1}	Order of 10^2-10^3 km s^{-1}
Mass derived from virial theorem (see §4)	Order of $10^{15}\ \mathcal{M}_{\odot}$	Order of 10^{12}-$10^{14}\ \mathcal{M}_{\odot}$
Other characteristics	Cluster often centered about one or two giant elliptical galaxies	
Examples	Coma cluster (No. 1656); CrB cluster (No. 2065)	Local Group, M81 group, Virgo cluster, Hercules cluster (No. 2151)

3.2. Galaxian Contents of Clusters

As already stated, spiral and irregular galaxies appear to be rare or lacking in regular clusters, but are common in the irregular ones. The Local Group, although a poor irregular cluster, is the one for which the most complete census exists. It contains three spiral galaxies, with M_v in the range -19 to -21, four irregular galaxies ($-14 \geq M_v \geq -18$), [(1)] four intermediate ellipticals ($-14 \geq M_v \geq -17$), [(2)] and up to nine dwarf ellipticals of the Sculptor type ($-9 \geq M_v \geq -14$), including three suspected companions of M31 discovered by van den Bergh (1972). There are no giant ellipticals that are certain members of the Local Group, but evidence from the rich clusters suggests that the luminosity function of elliptical galaxies increases with increasing absolute magnitude and giant ellipticals are rare; perhaps we should not expect to find one in a sample of only 17 to 20 galaxies. Undiscovered members of the Local Group may well exist, especially in the direction of the Milky Way. In particular, there are probably galaxies of lower luminosity than the Ursa Minor system (the least luminous known member of the Local Group). Remote globular clusters are known to exist, for example, at distances up to 10^5 pc (Abell 1955); if such systems are distributed uniformly throughout the Local Group, and are not just outlying members of our own Galaxy, they can properly be classed as galaxies themselves, as Zwicky (1957) and others have proposed. There is no certainty that the Local Group does not contain ever larger numbers of objects of ever smaller mass, ranging down to individual stars.

The irregular Virgo cluster displays a wider range of galaxian types. The majority of the brightest galaxies are spirals; 205 Shapley-Ames galaxies lie in the region of the Virgo cluster: 68 percent are spirals, and only 19 percent are ellipticals, the remainder being irregular or unclassified. The incidence of ellipticals in the Virgo cluster, however, is still significantly higher than among the brightest galaxies in the field. Moreover, the fraction of cluster galaxies that are elliptical increases at fainter magnitudes; of the 272 brightest galaxies in a 6.6° × 6.6° region centered at (1950) $\alpha = 12^h25^m$, $\delta = +13°$, 53 percent have been classified by the writer as spirals and irregulars, and 47 percent ellipticals and S0's. According to Shapley (1950), just under half of the Virgo galaxies are spirals in the magnitude range 12-14, and only about a quarter in the range 14-16. Reaves (1956, 1967) has searched for faint galaxies in the Virgo cluster on plates taken with the Lick 20-inch astrograph. Of about 1000 objects that Reaves considers possible or probable cluster members, he describes 76; only 11 of these are definitely not of the elliptical type, and cluster membership of some of the 11 is in doubt. The majority of Reaves's objects appear to be similar to IC 3475. They are probably elliptical galaxies of intermediate to low luminosity, ranging from objects like NGC 221 to the brightest Sculptor-type systems in the Local Group; however, Reaves (1962) emphasizes that this interpretation must be viewed with caution until accurate colors are available. Faint dwarf ellipticals (like the Draco and Ursa Minor systems) might be expected to exist in still greater numbers in the Virgo cluster, but they would not have been detected in Reaves's survey. Elliptical galaxies slightly brighter than Sculptor-type dwarfs have also been observed in some other nearby clusters, in particular the M81 group and the Fornax cluster (Hodge 1959, 1960; Hodge, Pyper, and Webb 1965). The scanty evidence available, therefore, suggests that dwarf ellipticals are very common in nearby irregular clusters, and could well be the most numerous kind of galaxy in all clusters.

Whereas spirals are common among the brighter galaxies in irregular clusters, and are actually in the majority among the bright Virgo galaxies, the very brightest cluster members tend to be giant ellipticals - unless the cluster population is small. The four brightest members of the Virgo cluster, according to Holmberg (1958), are the giant ellipticals NGC 4472 (m_{pv} = 8.5), NGC 4486 (m_{pv} = 8.7), NGC 4649 (m_{pv} = 9.0), and NGC 4406 (m_{pv} = 9.2). If the distance modulus of the cluster is 31.1 (Sandage 1968), these galaxies have absolute visual magnitudes in the range -21.9 to -22.6. Similarly, Morgan (1961) found that the brightest members in each of the 20 nearest clusters in the Abell catalog tend to have stellar populations of the "evolved" type - yellow giant stars.

Frequently one or two particularly luminous giant elliptical galaxies will be found near the center of a regular cluster; sometimes these "supergiant" ellipticals will exceed the brightness of the next most luminous cluster members by more than a magnitude. The center of the Coma cluster, for example, lies roughly midway between the giant ellipticals NGC 4874 and NGC 4889 [= NGC 4884]. More than 30 clusters containing supergiant galaxies as their brightest members (Morgan cD galaxies) have been described by Matthews, Morgan, and Schmidt (1964) and by Morgan and Lesh (1965). Bautz and Morgan (1970; Bautz 1972) classify clusters I to III according to their brightest elliptical galaxy. A type I cluster contains an extraordinarily large luminous cD galaxy that dominates the cluster, as does NGC 6166 in Abell cluster 2199. The Virgo and Corona Borealis (A2065) clusters, on the other hand, represent type III, in that each has no member that stands out noticeably against the giant galaxy background.

As stated above, the rich regular clusters, in contrast to the irregular ones, appear to be nearly or completely devoid of spiral and irregular galaxies. There are actually several spirals in the field of the Coma cluster, the nearest of the regular clusters. Whether or not they are cluster members, however, deserves discussion. The writer was able to find 47 galaxies that could definitely be classed as spirals on 48-inch Schmidt photographs covering a 70-square-degree region in the vicinity of and including the cluster. The radial velocities of some of these spirals are known, and most lie within 2000 km s^{-1} of the mean cluster velocity of 6866 km s^{-1}. Rood, Page, Kintner, and King (1972) treat these objects as cluster members. Subsequently Rood (1974) has analyzed the radial distribution of 30 spirals in the Coma field and concludes that many are probably members of the cluster, and that spirals and irregulars make up 15 percent of the Coma galaxies in the interval of the brightest 2.7 mag. Rood similarly investigated spirals near cluster A2199, and finds that they are probably field galaxies. The detailed distribution of spirals in and around other more remote regular clusters has not yet been investigated. In Section 5.2 we review the strong evidence for superclustering. Quite possibly the spirals in and near a regular cluster (e.g., Coma) share membership in a larger cloud of galaxies but are not properly part of the main condensation of the cluster. The hypothesis cannot be ruled out that at least the inner regions of regular clusters are completely lacking in spirals.

Spitzer and Baade (1951) suggested that the absence of spiral galaxies in rich clusters may be a result of collisions between the cluster galaxies, and the consequent removal of interstellar matter from them. In the Coma cluster, for example, a typical galaxy moves with a speed of the order of 10^3 km s^{-1} with respect to the cluster center; in 5×10^9 years such a galaxy would traverse a distance of 5×10^6 pc. At the time of the Spitzer-Baade analysis it appeared that this was several times the diameter of the cluster, and that most galaxies in passing back and forth through the cluster would have suffered at least one collision since the cluster was formed. With modern estimates of the extragalactic distance scale, however, and the correspondingly larger cluster diameters, it is not certain that the Spitzer-Baade mechanism can have effectively removed interstellar matter from most or all galaxies in a typical rich cluster unless its age is much greater than that usually assumed for the Universe. It is possible, therefore, that spiral galaxies either were never formed in the regular clusters, or have disappeared through other evolutionary processes.

Rich clusters of galaxies also tend to contain strong radio sources. Mills (1960) compared the positions of 1159 sources found in the Sydney survey with those of the 877 rich clusters in the Abell catalog that appear in the same survey region. He found 55 coincidences, whereas only 16 coincidences would be expected by chance. Van den Bergh (1961a) similarly compared positions of 282 sources in the 3C catalog with galactic latitudes greater than 25° with positions of clusters in the Abell catalog and found 27 coincidences, whereas nine would have been expected by chance. Moreover, van den Bergh finds that the radio magnitudes of the sources are correlated with the distances of the clusters, further strengthening the significance of the association of sources and clusters. More recently, Pilkington (1964) has rediscussed the coincidences between positions of clusters and of sources in the Cambridge and Sydney catalogs, and has added a search for such coincidences among the sources in the partially complete 4C survey. He finds that the radio positions tend to lie near the projected centers of clusters, and he gives a table of 41 sources that lie (in projection) within about 1 Mpc of cluster centers. He estimates that about 8 of these are chance associations. Rogstad, Rougoor, and Whiteoak (1965) have searched for 21-cm continuum radiation from 39 nearby clusters with the Caltech radio interferometer at Owens Valley, and have detected significant flux from 25 of them. They estimate that nearly 60 percent of the clusters listed in the three nearest distance categories in the Abell catalog contain detectable sources. The small angular sizes of the sources observed suggest that the radiation comes from individual galaxies, and not from sources spread over large intracluster spaces.

According to Minkowski (1963) and Matthews, Morgan, and Schmidt (1964), more than a third of the individual galaxies identified with radio sources are in rich clusters in the Abell catalog. On the other hand, Minkowski estimates that only about 10 percent of all galaxies are in clusters as rich as those catalogued by Abell. Van den Bergh (1961b) independently arrived at a similar estimate. Both van den Bergh (1961a) and Pilkington find that the number of coincidences between source and cluster positions are not correlated with cluster richness in the manner expected if collisions between galaxies were responsible for the radio emission. On the other hand, Rogstad and Ekers (1969) find that E and S0 galaxies in the field are as likely to be strong radio sources as are those in clusters; their data suggest that the propensity of clusters to be radio sources may simply reflect the tendency for E and S0 galaxies to be in clusters.

Clusters also tend to be X-ray sourg src="../../New_Gifs/geq.gif" alt="geq"> $M_v \geq$ - 18) - e.g., NGC 147, IC 1613, and NGC 205 - are often referred to as dwarfs; but since galaxies of lower luminosity appear to be even more common, here we shall use the term "dwarf" for the latter. Back.

[2] Back. See n. 1.

3.3. The Luminosity Function and Colors of Cluster Galaxies

Hubble (1936b) investigated the distribution of absolute magnitudes of 134 spiral and 11 irregular galaxies (most of them not in conspicuous clusters) and represented their luminosity function with a Gaussian curve having a mean absolute photographic magnitude of -14.19 and a standard deviation of 0.85 mag (the mean should be several magnitudes brighter to correspond to the modern distance scale). Hubble's luminosity function applies at best to the type of galaxies represented in his sample (late spirals and irregulars were chosen because in them he could identify what appeared to be individual stars, and hence could estimate distances), and there is no justification for assuming that it holds for elliptical galaxies, which are the major constituents of rich clusters, or, therefore, for cluster galaxies in general. In particular, the great prevalence of dwarf ellipticals was unknown to Hubble at the time of his pioneering investigation. We should, in fact, expect the luminosity functions to vary even among clusters of different morphological type and richness. Considerable confusion has resulted from the failure of some investigators to recognize the inapplicability of Hubble's luminosity function to clusters of galaxies.

Zwicky (1942a) may have been the first to summarize the observations favoring the existence of large numbers of galaxies of low luminosity. He also advanced quasi-statistical-mechanical arguments that the gal increasing magnitude. His argument is based on the assumption of a stationary universe in statistical equilibrium, and on the application of the Boltzmann principle to determine the relative numbers of galaxies in clusters and in the field, as well as the relative numbers of stars inside and outside galaxies. The analysis leads to the result that most stars (and other matter) should be in the intergalactic space; moreover, he believed that collisions should result in the fragmentation of some galaxies into smaller parts. Zwicky concludes (italics are his): "*...individual stars, multiple stars, open and compact star clusters and stellar systems of increasing population will be found in numbers presumably decreasing in frequency as the stellar content of the systems in question increases.*"

Few modern investigators would accept the assumption on which Zwicky's conclusions are based; however, his supposition that the galaxian luminosity function rises at faint absolute magnitudes is almost certainly correct - at least for galaxies in rich clusters to the magnitude limits observed. Later, from a rather ingenious analysis of the mean numbers of galaxies within clusters of various angular diameters, Zwicky (1957) derived the following expression for the integrated luminosity function of cluster galaxies:

$$N(\Delta m) = k(10^{\Delta m/5} - 1), \qquad (1)$$

where $N(\Delta m)$ is the number of galaxies in the range Δm between magnitude m and the magnitude of the brightest cluster galaxy. Unfortunately, as Zwicky defines his cluster angular diameters, they are not related to distance in the manner he assumes in the derivation of equation (1), as has been shown by the writer (Abell 1962) and by Scott (1962). Nevertheless, Zwicky's formula may be qualitatively correct, at least at the fainter magnitudes. He states (Zwicky 1957) that equation (1) is consistent with counts of galaxies in several clusters to two or more different magnitude limits (on plates with different exposure times or taken with different telescopes), and also with the distribution of magnitudes obtained with schraffier photometry among the brightest galaxies in several clusters (e.g., Zwicky and Humason 1964a, b).

The published data have been reexamined to determine the galaxian luminosity function by Kiang (1961) and by van den Bergh (1961b). Kiang, from magnitudes of galaxies both in clusters and in the field, adopts Zwicky's form of the differential luminosity function $\phi(M)$ (derivative of eq. [1]) for faint magnitudes, but finds that the function rises more sharply for bright magnitudes; he adopts a cubic law for $\phi(M)$ over the interval of the brightest 2.5 mag. Van den Bergh analyzes the absolute magnitudes of 240 bright field galaxies, of which 48 are ellipticals and 192 are spirals and irregulars. Although the magnitude range considered by van den Bergh is small (4.5 and 6.5 mag for ellipticals and spirals, respectively), his luminosity functions (found separately for ellipticals and for spirals and irregulars) are both in qualitative agreement with Zwicky's formula at faint magnitudes; however, they rise more rapidly at their bright ends.

Derived luminosity functions are, unfortunately, sensitive to photometric procedures, the difficulty of which are often not appreciated. The surface brightness of an elliptical galaxy drops off very slowly at large distances from the center of its projected image; Hubble's (1930) representation of the brightness is, in fact, an inverse-square function of the radial distance. Hubble's interpolation formula, if extended to infinite radius, leads to an infinite luminosity for a galaxy; thus at some distance the galaxian surface brightness must begin to drop more rapidly. Some investigators (Dennison 1954; Liller 1960; de Vaucouleurs 1948) have found finite luminosities (or "total" magnitudes) for elliptical galaxies, but their measures of surface brightness deviate from Hubble's law only at very large radial distances, where the measurements are extremely difficult. For a few elliptical galaxies, convergence of the total luminosity has not been ascertained observationally; photoelectric measures of M87 by Baum (1955), for example, show no evidence of deviation from Hubble's inverse-square law even to very great distances from the center. Even in spiral galaxies, where the spiral arms seem more sharply defined, the contribution to the total luminosity of the Population II coronal components, which may extend far beyond the spiral arms, is not accurately known.

"Total" magnitudes of galaxies are thus not easy to define unambiguously, and investigators should exercise considerable caution in interpreting published magnitudes. Consistent results have been obtained by Sandage (Humason, Mayall, and Sandage 1956), who defines the magnitude of a galaxy according to the total light contained within a given standard isophote, and by Holmberg (1950, 1958), who integrates the light over a galaxian image from several microphotometer tracings across it. Sandage's magnitudes, however, do not correspond to the same fraction of light in galaxies of similar size but different surface brightness (Abell 1962), and with Holmberg's procedure the contribution to the total light of a galaxy from very faint outer extensions that are below the photographic threshold remains unknown.

On the other hand, if the surface brightnesses of galaxies can be represented by a single model, "total" magnitudes for them can be operationally defined. It appears possible to find a satisfactory representation of the distribution of surface brightness in bright elliptical galaxies. Hubble's formula for the brightness I at a distance r from the center along the major axis of the projected image of an elliptical galaxy,

$$\frac{I}{I_0} = \frac{1}{(1 + r/a)^2} \tag{2}$$

(where I_0 and a are parameters for a particular galaxy), describes the surface brightness over most of its observable image very well. De Vaucouleurs's (1948) formula is

$$\log(I/I_e) = -3.33[(r/r_e)^{1/4} - 1], \tag{3}$$

where r_e is the distance from the center along the major axis to the isophote within which one-half of the total light is emitted, and I_e is the surface brightness at that isophote. It agrees well with the Hubble law for small r, and has the advantage of leading to finite total magnitudes that are in satisfactory agreement with those of Sandage and Holmberg.

The writer has found that a convenient interpolation formula for the distribution of light in an elliptical galaxy is a modification of the Hubble law:

$$\begin{aligned} \frac{I}{I_0} &= \frac{1}{(1 + r/a)^2} \quad \text{for } r/a \leq 21.4 \\ \frac{I}{I_0} &= \frac{22.4}{(1 + r/a)^3} \quad \text{for } r/a > 21.4. \end{aligned} \tag{4}$$

Equation (4) agrees with de Vaucouleurs's formula to the precision of present-day photometry, and leads to "total" magnitudes that are (statistically) close to those of Sandage and Holmberg. The writer and Mihalas (Abell and Mihalas 1966) have developed a technique of determining the parameters I_0 and a, and hence the "total" magnitude, of an elliptical galaxy from measures of the brightness of its extrafocal image on each of two or more calibrated photographs taken different amounts out of focus. The total magnitude obtained is actually the magnitude of a fictitious galaxy with surface brightness given by equation (4), and that has extrafocal images of the same brightness as the measured galaxy. The precise form of the surface brightness law is not important except to establish the zero point of the magnitude scale, for in practice it is rare that contributions to an extrafocal image come from parts of a galaxy for which r/a exceeds 21.4. The magnitudes obtained may not always be accurate for individual galaxies, for all elliptical galaxies may not be built on the same model; but they are statistically self-consistent, and are free from systematic effects that depend on the distance of a galaxy measured; in particular, they are suited to the study of luminosity functions of elliptical cluster galaxies, and to the comparison of luminosity functions of different clusters.

To date, the writer has applied the procedure to determine the luminosity functions of six clusters, of which four are regular clusters of known redshift. The luminosity functions of these clusters, which consist predominantly of elliptical galaxies, are all similar. If the logarithmic integrated luminosity functions, log $N(\Delta m)$, are plotted, they can all be fitted together very satisfactorily with horizontal and vertical shifts that depend only on the relative cluster distances and richnesses. Such a combined integrated luminosity function is shown for four rich clusters in figure 3; they are cluster numbers 1656 (Coma), 2199 (surrounding NGC 6166), 151, and 2065 (Corona Borealis). The coordinates of figure 3 have arbitrary zero points. The smooth curve labeled "Abell" is adopted as the mean function, log $N(\Delta m)$, for the four clusters. The dashed curve labeled "Zwicky" corresponds to Zwicky's integrated luminosity function, defined by equation (1).

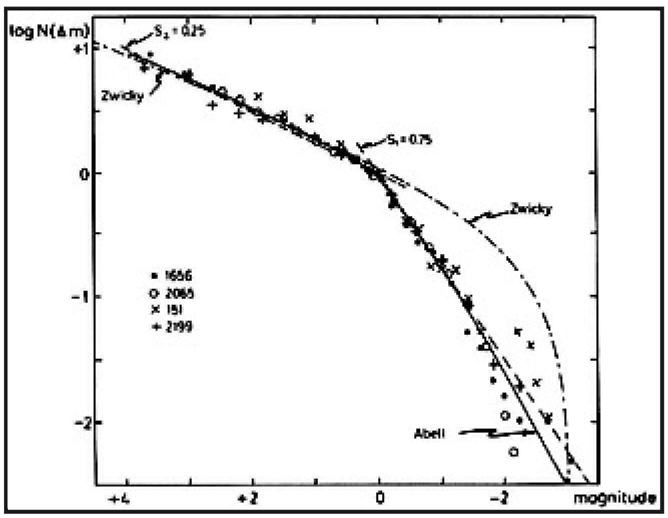

Figure 3. The logarithmic integrated luminosity functions of four rich clusters. The symbols and curves are explained in the text. The zero points of the ordinates and abscissae are arbitrary.

Curves of log $N(\Delta m)$ compiled from published data of several other observers are plotted in the same manner in figure 4. The points for cluster 1377 (Ursa Major cluster No. 1) are from very old photometry by Baade (1928). The "Zwicky" data for cluster 1656 and the Virgo cluster are from the first two volumes of the *Catalogue of Galaxies and Clusters of Galaxies*. Only the central part of the Virgo cluster is used, and the *Catalogue* data covers only the interval of the brightest 2.7 mag in cluster 1656. The "Holmberg" data for the Virgo cluster are from Holmberg (1958). For comparison, there is also shown the logarithmic integrated luminosity function for 48 elliptical field galaxies, taken from van den Bergh (1961b). It must be emphasized that the photometric procedures differed among the various observers; large zero-point differences between the different magnitude systems are expected, and scale errors may exist as well. Zero-point and scale errors are especially likely in the early photographic photometry of Baade. It should also be noted that the Virgo data include many spirals, and the apparently excellent agreement between the Virgo luminosity function and that of the other clusters may be fortuitous. The "Abell" and "Zwicky" curves in figure 4 are the same ones as in figure 3.

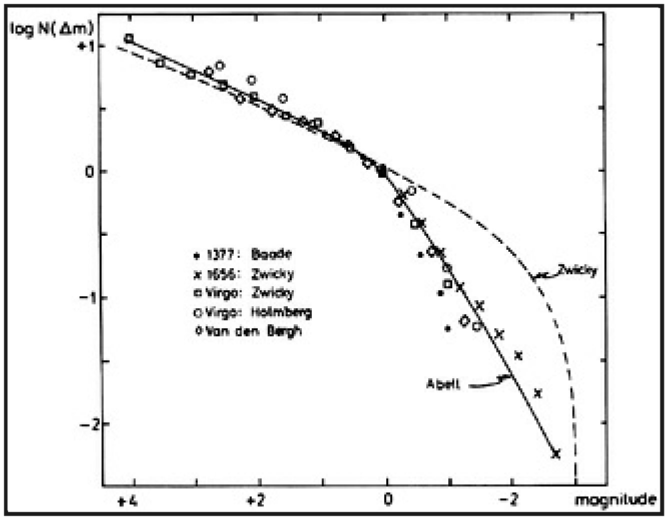

Figure 4. Cluster logarithmic integrated luminosity functions adapted from the published data several observers. See text for explanation. The scale of the abscissae and ordinates is as in fig. 3, and the zero points are arbitrary.

The differential luminosity function $\phi(M)$ corresponding to the "Abell" curve in figures 3 and 4 is shown in figure 5. The ordinates are arbitrary; the abscissae are M_{pv}, determined by fitting van den Bergh's luminosity function to the Abell cluster luminosity functions (van den Bergh's absolute magnitudes have been adjusted to correspond to a Hubble constant of 75 km s^{-1} Mpc^{-1}). It appears that $\phi(M)$ for elliptical galaxies can not be represented by a single exponential function. The function log $N(\Delta m)$ shows a very definite change of slope after the interval of the brightest 2 to 3 mag. This abrupt change of slope corresponds to the maximum near the bright end of $\phi(M)$. There is some evidence that at least in the Coma cluster this peak is contributed mainly by galaxies near the central core of the cluster (Rood 1969; Rood and Abell 1973).

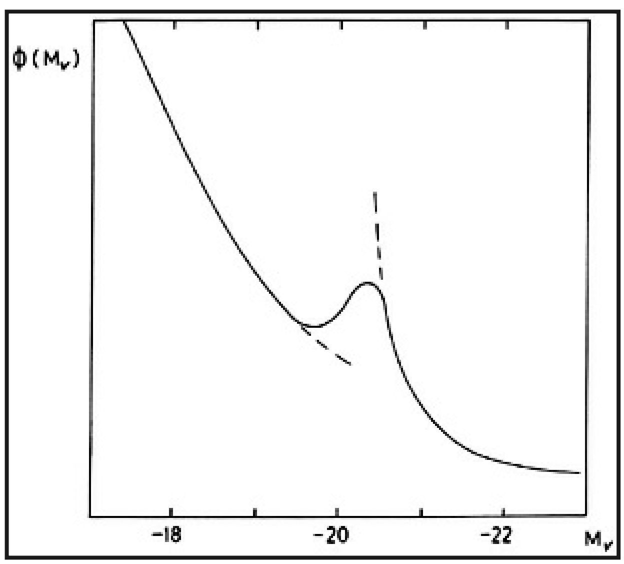

Figure 5. The differential cluster luminosity function, derived from figs. 3 and 4. The scale of the ordinates is arbitrary.

The composite luminosity function for the clusters in figure 3 can be represented rather closely by two intersecting straight lines (*dashed lines* in fig. 3), which represent simple exponential relations over two different magnitude intervals. These lines are merely interpolation formulae, and the approximately exponential character of the integrated luminosity functions should not be taken too literally. For example, a simple exponential function can hardly be expected to match the very brightest galaxies in different clusters of different Bautz-Morgan types (some of which have supergiant cD galaxies and some of which do not). Moreover, if the two straight lines were to apply rigorously the differential luminosity function would have to be discontinuous (*dashed lines* in fig. 5), which is, of course, physically unrealistic. Nevertheless, the magnitude at which the two lines intersect in figure 3, which we define as m^*, is an operationally defined point on the luminosity function. If all galaxies in all clusters are selected from the same population with a parent luminosity function like that in figure 3, m^* should be a useful "standard candle" for determination of relative distances of clusters.

The luminosity functions of all clusters investigated so far show the same characteristic shape (i.e., change of slope as in fig. 3). The best fit of the luminosity function of a cluster to the one in figure 3 determines m^* for that cluster, even if the photometric data are not very complete and if its luminosity function cannot be fitted unambiguously by the two exponential relations. For those clusters of known redshift and for which m^* has been so determined, the plot of log z versus m^* has a scatter of about 0.1 mag. (s.d.) (Abell 1962; Bautz and Abell 1973), significantly less than in the corresponding Hubble diagram in which the first brightest cluster galaxies are used as standard candles.

The luminosity function data used to construct figures 3 and 4 are summarized in table 2. The cluster numbers are those in the Abell (1958) catalog. The slopes, s_1 and s_2, of the straight lines used to approximate log $N(\Delta m)$ are defined by:

$$\begin{aligned} \log N(\Delta m) &= K_1 + s_1 m \quad (\text{for } m \leq m^*) \\ \log N(\Delta m) &= K_2 + s_2 m \quad (\text{for } m > m^*). \end{aligned} \tag{5}$$

Log $N(\Delta m^*)$ is the ordinate of the intersect of the two lines. Note that zero-point and scale differences may exist between the magnitude systems of the different observers, and that m^* is given in photovisual magnitudes for the clusters observed by the writer, and in photographic magnitudes for those observed by others. The slope s_2 is sensitive (particularly for distant clusters) to the corrections applied to the cluster photometry to take account of foreground and background field galaxies. The writer has not yet analyzed the field in a final way, and the values of s_2 given for the clusters investigated by him should be considered provisional. For the purpose of discussion he adopts for the mean luminosity function the values, $s_1 = 0.75$, and $s_2 = 0.25$. (In the limit of faint magnitudes, Zwicky's curve has a slope, $s_2 = 0.20$ - in reasonable agreement with the value adopted here.)

Table 2. Data pertaining to the luminosity functions of several clusters

Cluster	Mean Radial Velocity (km s^{-1})	Reference for Velocity	s_1	s_2	m^*	log $N(\Delta m^*)$	Observer, or Reference for Photometry
Virgo	1136	Humason et al. (1956)	0.72	0.20	m_{pg} = 11.9	1.15	Zwicky, Herzog, and Wild (1961)
Virgo			0.78	0.33	m_{pg} = 10.9	1.25	Holmberg (1958)
1656	6866	Lovasich et al. (1961)	0.75		$m_{pg} \geq$ 15.6	$\geq$ 2.25	Zwicky and Herzog (1963)
1656			0.78	0.25	m_{pv} = 14.7	2.25	Abell
2199	8736	Minkowski (1961)	0.75	0.26	m_{pv} = 15.4	2.00	Abell
1377	15269	Humason et al. (1956)	1.20	0.35	m_{pg} = 16.9	1.25	Baade(1928)
151	15781	Humason et al. (1956)	0.73	0.33	m_{pv} = 16.4	1.95	Abell
2065	21651	Humason et al. (1956)	0.72	0.27	m_{pv} = 17.2	2.23	Abell
Field ellipticals			0.95	0.27	M_{pg} = 19.5†	‡	van den Bergh (1961b)

NOTE. - According to de Vaucouleurs (1961b), the Virgo cluster is at least two clusters seen in projection. One component, consisting mostly of elliptical galaxies, has a mean radial velocity of 950 km s^{-1}; the other, consisting mostly

component, consisting mostly of elliptical galaxies, has a mean radial velocity of 950 km s^{-1}; the other, consisting mostly of spirals, has a mean radial velocity of 1450 km s^{-1}.
† For H = 75 km s^{-1} Mpc^{-1}.
‡ For H = 75 km s^{-1} Mpc^{-1}, log $N(\Delta M_{pg}{}^*)$ = -3.79 galaxies Mpc^{-3}.

Since figures 3 - 5 and table 2 were prepared for this review, considerable new data on cluster luminosity functions have become available. Gudehus (1971, 1973) has determined luminosity functions for clusters A754 and A1367, and has confirmed that of Abell for A2065. Abell and Mottmann (in preparation; see Bautz and Abell 1973) have obtained the luminosity function for cluster A2670. Eastmond (Abell and Eastmond 1968; Abell 1972) has new photometry for the elliptical galaxies in the Virgo cluster. Noonan (1971) presents counts in clusters A1656 and A2065 to different magnitude limits that are consistent with the luminosity functions for those clusters shown in figure 3. Rood and Baum (1967, 1968), while not discussing the luminosity function per se, do give very useful photometric and other data for 315 Coma cluster galaxies. Huchra and Sargent (1973) have derived a luminosity function for field galaxies from the data in the *Reference Catalogue of Bright Galaxies* (de Vaucouleurs and de Vaucouleurs 1964) that is also in excellent agreement with the cluster luminosity function in figure 3. Because the more recent data are all consistent with those presented here, the figures and table have not been updated.

The cluster luminosity function has not yet been determined observationally at magnitudes fainter than about $m^* + 4$. Galaxies at this observational limit have absolute magnitudes near $M_v \approx -16.5$; these are still relatively luminous objects, brighter, in fact, than all but four Local Group galaxies. Reaves (1966) searched for dwarf galaxies in the central core of the Coma cluster and found 32 probable dwarfs in the range $-14 > M_v > -16$. If the total content of dwarf galaxies in nearby groups and in the Virgo cluster is typical, we would expect the luminosity function to continue to increase at fainter magnitudes, but the precise form of $\phi(M)$ - for example, whether it increases monotonically - is not known. In particular, there is no justification for extrapolating the interpolation formula (eq. [5]) for the integrated luminosity function beyond the observed magnitude range. Such an extrapolation of equation (5) would predict that the number of Sculptor-type systems (with M_v in the range -9 to -14) in the Coma cluster is of the order 10^5. Nevertheless, even this large number of dwarfs would contribute only slightly to the total luminosity (and presumably also to the total mass) of the cluster; extrapolation of equation (5) to infinite magnitude leads to a finite luminosity for the cluster of 1.2×10^{13} times the solar luminosity, and only 13 percent of this light would be contributed by galaxies fainter than $m_{pv} = 18.3$, the magnitude limit for which the luminosity function is observed.

Colors of galaxies, at least of their inner portions, can be measured far more easily than total magnitudes, and fairly accurate data are available. Published results of photoelectric photometry by de Vaucouleurs (1961a) and photographic photometry by Holmberg (1958) include galaxies both in the field and in the Virgo cluster. Colors of the brighter cluster galaxies appear to be the same as those of galaxies of similar morphological type in the field. For giant ellipticals, $B - V$ color indices are about 0.95 to 1.00; $B - V$ decreases through the sequence of spirals from about 0.8 for Sa's to about 0.5 for Sc's. Both de Vaucouleurs (1961a) and Baum (1959), however, report that colors of elliptical galaxies in the Virgo cluster that are successively fainter than $m_{pv} \approx 11.5$ are successively bluer than those of giant ellipticals, and at $m_{pv} \approx 14$, $B - V$ is from 0.6 to 0.8. Sandage (1972) has obtained three-color observations of six dwarf ellipticals in the Virgo cluster, and of 25 elliptical galaxies in the Coma cluster, and confirms the correlation between $B - V$ and magnitude. He also shows that there is an even stronger dependence of $U - B$ on V, in the sense of fainter galaxies being bluer. On the other hand, both de Vaucouleurs's and Holmberg's data include color measures of several intrinsically fainter Local Group ellipticals of intermediate luminosity, which appear to have colors that are about the same as those of giant ellipticals. Holmberg even includes two dwarf ellipticals (the Leo systems in the Local Group), and they also have approximately "normal" colors.

3.4. Populations of Clusters

It is not possible to state with certainty whether particular objects in the field of a cluster of galaxies are actually members of that cluster. Even radial velocities do not provide an unambiguous answer because of the velocity dispersion that always exists within a cluster. Only statistically can we speak of cluster populations, and then we must define: (1) the magnitude range counted; (2) the way the cluster is presumed to be bounded; and (3) the way corrections are applied for the field of foreground and background objects (of course, the meaning of the "field" itself is open to question).

Populations are given by Zwicky and his collaborators in their *Catalogue of Galaxies and Clusters of Galaxies*. The population listed for a cluster is the number of galaxies visible on the red Palomar *Sky Survey* plate that are contained within the isopleth (drawn by eye estimate) at which the surface density of galaxies is twice that in the surrounding field; the cluster counts are corrected for the mean field count itself. It is clear that the populations are not independent of distance, since a greater magnitude range is counted in nearby clusters than in distant ones. Also, since an isopleth contains not all of its cluster, but only that part within which the surface density is twice that of the surrounding field, the entire cluster population is not counted, even in the magnitude range considered. Moreover, as has been shown by Abell (1962) and Scott (1962), the isopleth contains a smaller fraction of the projected image of a distant cluster than of a nearby one. Finally, Zwicky himself has called attention to the fact that his cluster populations may be affected by interstellar and possible intergalactic absorption. Having noted these various selection effects, we summarize the population listed in the first two volumes of the *Catalogue* in table 3. The data are segregated by Zwicky's morphological cluster classes (compact, medium compact, and open), and also according to his estimate of the relative cluster distances. The radial velocities corresponding to the distances separating the five distance classes - near, medium distant (MD), distant (D), very distant (VD), and extremely distant (ED) - are quoted as 15,000, 30,000, 45,000, and 60,000 km s^{-1}, respectively.

Table 3. Numbers of clusters of various populations in the first two volumes

Population	< 100	100-199	200-299	300-399	400-499	500-599	≥ 600
Near:							
Compact	0	2	2	2	0	2	6
Medium compact	6	27	27	9	8	3	14
Open	9	41	27	19	7	1	14
MD:							
Compact	1	9	10	3	2	2	1
Medium compact	25	120	48	26	14	7	10
Open	21	109	33	8	7	3	1
D:							
Compact	19	36	9	3	1	1	3
Medium compact	86	198	60	11	5	3	1
Open	50	164	24	5	1	0	0
VD:							
Compact	139	143	30	2	1	0	0
Medium compact	267	190	36	1	4	0	0
Open	95	97	12	0	0	0	0
ED:							
Compact	465	132	11	0	1	0	0
Medium compact	307	159	10	1	0	0	0
Open	36	18	0	0	0	0	0

Another body of data comes from the writer's cluster catalog (Abell 1958). This study concerns only the very richest clusters, but those are chosen in such a way as to comprise a relatively homogeneous sample, and the "populations" are defined in a manner to be as independent as possible of distance. The population of a cluster in the Abell catalog is the number of galaxies brighter than $m_3 + 2$, where m_3 is the photo-red magnitude of the third brightest cluster member. All counts were ade on the red *Sky Survey* plates, and include those galaxies in the prescribed magnitude range that are contained within a circle centered on the image of the cluster, minus the number in the same magnitude range contained within a circle of similar size in the nearby field. The circles used were large compared to the main concentrations of galaxies within the cluster. A circle radius, in millimeters, was $4.6 \times 10^5 / cz$, where cz, the velocity of recession in km s^{-1}, was estimated by comparing the tenth brightest cluster member with the tenth brightest members of clusters of measured redshift. The counts were thus extended to approximatually the apparent dependence of cluster richness on distance is not large, and is very sensitive to slight errors of observations that may also depend on distance. The possible role of such error(1964), who shows that the Just effect can be explained without evolution if Abell failed to include in the homogeneous sample only about 10 percent of the most distant clusters. Paál suggests that such a loss of remote clusters could result from the fact that the diameters of the counting circles used by Abell are inversely proportional to the cluster redshifts, rather than reflecting the dependence of angular diameter on distance that is predicted by any particular cosmological theory. It is the judgment of the writer that further independent observations are required to establish whether the Just effect is real.

Table 4. Numbers of clusters of various populations in the abell catalog

Population	clusters
50-79	1224
80-129	383
130-199	68
200-299	6
$\geq$ 300	1

The Abell and Zwicky data on cluster richnesses are not easy to compare because of the different ways in which cluster "populations" are defined. Both tables 3 and 4, however, show clearly that the numbers of clusters increase rapidly with decreasing population. It is unfortunate that data do not exist on the relative numbers of rich clusters and small groups; probably the latter are far more frequent. Multiplicity is common even among what appear to be "field" galaxies. According to Holmberg (1962a), only 47 percent of nearby systems have multiplicity 1 (that is, are single galaxies), and 53 percent are either double or multiple systems. It should be noted, however, that all of the systems investigated by Holmberg are within what may be a local supercluster (see Section 5); moreover, multiplicity exists even within irregular clusters.

3.5. Sizes and Structures of Clusters

Sizes of clusters are difficult to determine because (*a*) the boundary between a cluster and the field is indefinite, and (*b*) usually only the brighter cluster members are observed whereas if the cluster has approached a state of statistical equilibrium the fainter unseen members should be more broadly distributed in space. On the other hand, as discussed in the next section, there is not yet conclusive evidence that even a single cluster is in a state of statistical equilibrium; if we assume that the bright and faint members of a cluster do, in fact, occupy the same volume of space, and if we can distinguish cluster from field galaxies, then we can obtain an estimate of the cluster size. There is some justification for this assumption, at least for a few irregular clusters. Dwarf elliptical galaxies in both the Virgo cluster (Reaves 1956, 1964) and the Fornax cluster (Hodge, Pyper, and Webb 1965) occupy approximately the same area in the sky as the bright cluster members (although they do not necessarily have the same spatial distribution). The angular diameter of the Virgo cluster (Sandage 1958) is about 7°; the corresponding linear diameter is about 3×10^6 pc (for $H = 50$ km s^{-1} Mpc^{-1}). The irregular Hercules cluster (cluster 2151) has dimensions of about 3.8×10^6 by 2.5×10^6 pc (Burbidge and Burbidge 1959b). The Local Group is about 1×10^6 pc in diameter. The sizes of the rich regular clusters are more controversial (see below), but are probably of about the same order.

The sizes of the largest clusters in the *Catalogue of Galaxies and Clusters of Galaxies* are discussed by Zwicky and his collaborators (Zwicky and Rudnicki 1963; Zwicky and Berger 1965; Zwicky and Karpowicz 1965, 1966). Although there is considerable scatter in the data, they report that the largest clusters of all types (compact, medium compact, and open) have about the same size, which is of the order 10^7 pc. This is actually a lower limit for it is the diameter corresponding to Zwicky's isopleth where the surface density of galaxies is twice that of the field, and not to the entire clusters.

The irregular clusters do not display symmetry or regular structure. Multiple condensations of galaxies, however, are often present. A statistical study of the surface distribution of galaxies in seven rich clusters was made to estimate the extent of subclustering within them (Abell, Neyman, and Scott 1964). Counts of galaxies in rings centered on galaxies within the clusters are compared to counts in rings centered on arbitrary grid points. Higher mean counts in the rings surrounding galaxies than in those around the grid points (after correction for any central concentration of the cluster as a whole) indicates subclustering, and analysis of the counts gives information on the extent of the subclustering. Of the clusters investigated to date, all that are irregular show evidence of subclustering; the two most regular clusters (1656 and 2065) do not. The subgroups appear to have characteristic radii of the order 10^5 pc. There are, of course, two subgroups in the Local Group - centered on the Galaxy and on M31. It is also well known that double and multiple galaxies are common in certain other irregular clusters; Holmberg (1937) and van den Bergh (1960) have called attention to probable binary systems in the Virgo cluster; and the Burbidges (1959b), to such systems in the Hercules cluster.

The rich regular clusters, which show high central concentration and spherical symmetry, and little or no subclustering, are amenable to representation by theoretical models of galaxy distribution. Data on the surface distributions of galaxies in a fair number of clusters (both regular and irregular) are now published, although the completeness, quality, and format of the data vary considerably from cluster to cluster. Fortunately some clusters have been studied independently by several investigators; comparison of their results can be a valuable aid in assessing the reliability of the available material. Some references to the distribution of galaxies in clusters (listed by number in the Abell catalog) are:

31	N. Bahcall (1972)
234	Zwicky (1956)
426 (Perseus)	Zwicky (1957); Rudnicki (1963)
732	Zwicky (1956)
801	Zwicky (1956)
1060 (Hydra I)	Zwicky (1957); Kwast (1966)
1132	N. Bahcall (1971)
1185	Rudnicki and Baranowska (1966b)
1213	Rudnicki and Baranowska (1966b)
1367	Rudnicki and Baranowska (1966a)
1643	Zwicky (1956)
1656 (Coma)	Zwicky (1937, 1942b, 1957, 1959); Omer, Page, and Wilson (1965); Noonan (1971); Rood et al (1972); N. Bahcall (1973b)
1677	Zwicky (1956)
2065 (Corona Borealis)	Zwicky (1956); Noonan (1971)
2199	Clark (1968); Rood and Sastry (1972); N. Bahcall (1973c)
Cancer	Zwicky (1957)
Pegasus	Zwicky (1957)

Counts in six clusters by the writer are as yet unpublished, except for a summary of the counts in cluster 1656 by Noonan (1961a, b). The counts of galaxies on the Lick astrographic plates by Shane and his collaborators reveal many conspicuous clusters, and isopleths for several of these have been published separately by Shane (1956b).

Several types of analytical representation of the galaxy distribution in rich clusters have been attempted. Zwicky has fit the projected distribution of galaxies in several clusters to a bounded isothermal distribution projected on a plane. An early solution for the density distribution in the isothermal polytrope was by Emden (1907), who tabulates the density as a function of the dimensionless variable ξ, representing the radial distance from the center of the configuration. Zwicky relates the linear distance r from the center of a cluster of galaxies to Emden's ξ by $r = \alpha \xi$, and calls the scale factor α the *structural index* of the cluster. The values he reports for α (1957), translated to the distance scale used in this review (H = 50 km s^{-1} Mpc^{-1}), are 24.2, 25.0, 32.3, and 13.8 (in units of 10^{22} cm) for clusters 1656, 1060, 426, and the Cancer cluster, respectively. Zwicky cites the rather small spread in the values for α among different clusters (actually, the spread is about a factor of 2) as evidence that all regular clusters are similar in size and structure. The writer's counts in cluster 1656 have also been compared with the isothermal polytrope, both by himself (unpublished) and by Noonan (1961a, b). The isothermal representation is a satisfactory one, but the writer finds $\alpha = 48 \times 10^{22}$ cm - almost twice Zwicky's value. More recently N. Bahcall (1972, 1973a, c) has shown that the galaxy distribution in several clusters can be matched satisfactorily to the isothermal polytrope, and she suggests further that the structural index (or an equivalent scaling factor) can be used, statistically, as a distance indicator for clusters.

In distant clusters of small angular size, Zwicky (1956) explains that α is difficult to determine with precision. For such remote systems, he introduces another parameter, which he calls the *distribution index*, DI, defined by

$$\mathrm{DI} = \frac{N(a_0)}{100N(a_0/10) - N(a_0)}, \qquad (6)$$

where $N(a_0)$ and $N(a_0 / 10)$ are the total numbers of cluster galaxies within angular distances a_0 and $a_0/10$ of the cluster center, respectively, after subtraction of n_a galaxies for each square degree. The quantity n_a is the mean number of galaxies per square degree along the circle of radius a_0, and thus represents an upper limit to the density of field galaxies. Zwicky chooses a_0 to be inversely proportional to the redshift of the cluster, so that it presumably corresponds to the same linear distance from the center in a nearby as in a remote cluster; for a cluster of redshift 51,700 km s^{-1}, a_0 has the value 10'.0. Zwicky attempts to adjust the exposure times of the cluster photographs so that the counts are extended through a similar magnitude range in all of them. If there were no central concentration of galaxies in a cluster, DI would be infinite. Actual values of DI for seven clusters are given in table 5, adapted from Zwicky (1956). The observed velocity of recession, *cz*, for each cluster is given, and also $N(a_0)$, which Zwicky points out is a lower limit to the total cluster population. Counts in clusters 1656 and 2065 were made with the 48-inch Schmidt telescope, and Zwicky states that it is difficult to adjust them to those made in the five more remote clusters on 200-inch telescope plates. Nevertheless, the small spread in the values of $N(a_0)$ and DI suggest a remarkable similarity among the regular clusters investigated.

Table 5. Minimum populations and distribution indices in seven regular clusters of galaxies*

Cluster (Abell Number)	cz (km s^{-1})	$N(a_0)$	DI
234	51,700	461	0.115
732	61,000	451	0.110
801	57,600	391	0.129
1643	59,300	394	0.115
1656	6,850	467	0.084
1677	54,900	332	0.107
2065	22,000	551	0.103

* Adapted from Zwicky 1956.

Other investigators have experimented with different mathematical representations of the density distribution in clusters of galaxies. De Vaucouleurs (1960) and Shane and Wirtanen (1954) find that the surface density of galaxies in the Coma cluster (1656) follows closely a law of the form

$$\log N(r) = ar^{1/4} + b, \qquad (7)$$

where $N(r)$ is the projected density at distance r from the center and a and b are constants. Omer, Page, and Wilson (1965) have averaged counts of galaxies in parallel strips across the same cluster by Omer, Page, and Wilson and by Shane and have represented the counts by a series of Hermite polynomials of even order, from which they derived a formula for the spatial density of galaxies in the cluster. Scott (1962) represents Zwicky's counts in the five most distant clusters listed in table 5 by Maxwellian distributions for which the ratio of σ to the distance of each cluster is 4×10^{-4}; for all five clusters, the linear value of σ is about 4.5×10^5 pc. All of these mathematical distributions seem to fit actual clusters about as well as the isothermal one. Each also involves a fitting constant or scale factor, such as α, DI, or σ; and if, as seems possible, the rich regular clusters are similar to one another, the relevant scale factor might prove valuable as a criterion for cluster distances - at least statistically.

An observation important to the study of the dynamics of a cluster is to determine whether there is a segregation of its bright and faint galaxies. If it has reached a state of equipartition of energy, the fainter - presumably less massive - galaxies should fill a larger volume of space than the brighter ones. Zwicky (1942b) presents evidence that such a segregation exists among the brighter Virgo cluster galaxies; but there is also a segregation according to galaxy type, the spirals showing a larger radial distribution than the ellipticals. Both Holmberg (1962b) and de Vaucouleurs (1961b) have shown that the mean radial velocities are different for the spirals and ellipticals in the cluster. De Vaucouleurs argues a strong case that the Virgo cluster is really (at least) two clusters seen in projection - one being a compact cluster of elliptical and S0 galaxies, and the other a loose open cluster consisting mostly of spirals and irregulars. If de Vaucouleurs's interpretation is correct, then differences in the radial distributions of Virgo cluster galaxies of different types and/or magnitude cannot be regarded as evidence of segregation.

Rudnicki (1963), who has counted galaxies in cluster 426, believes that in that cluster the faint galaxies show a wider surface distribution than the bright ones. Unfortunately, one side of cluster 426 is partially hidden by interstellar absorption, and Rudnicki may have adopted too low an estimate for the density of galaxies that are in the field; a moderate and entirely possible increase in the field estimate completely removes the apparent segregation of bright and faint galaxies in that cluster.

Hodge, Pyper, and Webb (1965) have compared the distribution of the 50 faint galaxies they discovered in the Fornax cluster to that of the brighter cluster members. They find that cluster membership is too difficult to ascertain farther than 4° from the center, so they limit their comparison to the inner regions. There, the faint galaxies appear to show less central concentration, suggesting that a real Segregation exists. Unfortunately, the number of bright galaxies is small (about 20), and small sample fluctuations or subclustering may influence the picture.

The fact that the peak at the bright end of the luminosity function of the Coma cluster seems to be at least partially due to brighter galaxies at the cluster center (Rood 1969; Rood and Abell 1973) indicates some dependence of the galaxy distribution in the cluster on galaxian mass. N. Bahcall (1972, 1973c) finds some possible segregation by mass of galaxies in cluster A31, but no evidence for the same in cluster 2199. Kwast (1966) finds the bright and intermediate galaxies in A1060 to have the same distribution, but the faint galaxies in the cluster to be more broadly distributed. Rudnicki and Baranowska (1966a, b) find some evidence for mass segregation in A1185 and A1213, but not in A1367.

The best studied cluster, and the best example for discussion, is the Coma cluster (1656). Here again, different investigators disagree on the size of the cluster and the distribution of bright and faint galaxies. Zwicky (1957) holds that the cluster is at least 6° in radius, that it contains about 10^4 members brighter than $m_{pg} = 19$, and that there is marked segregation of its brighter and fainter members. Noonan (1961a, b), however, has reanalyzed Zwicky's own published counts and finds the cluster to be only 100' in radius, and to have identical distributions of bright and faint galaxies. Omer, Page, and Wilson (1965), from the analysis of their own and of Shane's counts in the Coma cluster, arrive at the same conclusion as Noonan. These different conclusions result from different interpretations of the surface density of field galaxies. The mean field density is quite constant from 2° to 6° from the cluster center, but is slightly lower in some directions at a distance just beyond 6°. Zwicky interprets the lower average density 6° to 7° from the cluster as the true field and attributes the excess of faint galaxies at distances less than 6° to cluster membership. The other investigators interpret the higher density from 2° to 6° as the correct field, in which case there is not an excess of faint galaxies with a larger radial distribution than the bright ones; the total cluster population to $m_{pg} \sim 19$ is then only about 10^3.

The writer (Abell 1963) finds the cluster to be slightly elongated in a northeast-southwest direction, and to be probably contained within an oval region of dimensions 5.4° × 4.3°. The oval shape of the cluster is also apparent in the equal-density contours of Shane and Wirtanen (1954), and in unpublished data of Reaves (Omer, Page, and Wilson 1965). To the magnitude limit $m_{pv} = 18.3$, he has found that in a 70 square-degree area outside this oval region the number of galaxies brighter than magnitude m is given by $\log N(m_{pv}) = \text{constant} + 0.6 m_{pv}$, which is expected if these are field galaxies distributed at random through space. At most, only a few percent of these galaxies can be faint outlying cluster members. Thus, the cluster cannot contribute appreciably to the field beyond the 5.4° × 4.3° region. The writer adopts 77 galaxies per square degree (with $m_{pv} \leq 18.3$) as a lower limit to this field density; the correct value may be a little higher, because the average is taken over an area which includes the low-density region several degrees away from the cluster center. If the counts within the oval cluster region are corrected for this field density, the luminosity function obtained for the cluster is the one given in table 2. The radial spatial distributions of galaxies brighter than $m_{pv} = 16.0$ and in the range $m_{pv} = 16.0 - 18.3$, derived from averaged counts in north-south and east-west strips across the cluster, are shown in figure 6. The slight segregation of the brighter and fainter galaxies that appears to be exhibited in figure 6 may result from the adoption of slightly too low a field density; this, in other words, is an upper limit to the segregation of bright and faint galaxies. The writer's results are not incompatible with those of Noonan and of Omer, Page, and Wilson, who find no segregation at all, and a total cluster radius of 100', or 4×10^6 pc. In addition, they are compatible with results of N. Bahcall (1973b), who finds no evidence for segregation of bright and faint galaxies except possibly for a slightly (20 percent) higher concentration of the brightest cluster galaxies near the core.

Rood et al. (1972) also concur that there is no marked segregation of bright and faint galaxies in the Coma cluster. However, they find the cluster to have a radius of 200'. The writer thinks it possible that the larger size Rood et al. find may result from their treating as members some outlying galaxies that may be part of a superstructure of galaxies that do not really belong to the main Coma cluster concentration. However, this question bears on how a cluster is defined in the first place, and there exists no unambiguous agreed-on operational definition. As remarked earlier, answers to such questions as the memberships, sizes, and distributions of galaxies of different masses within clusters depend on how clusters are defined and how their members are distinguished from the field. Differences between how these matters are handled may well account for the diversification of results described in this section.

3.6. Velocity Dispersions in Clusters

Radial-velocity observations have been made in a number of clusters, especially to obtain data for cosmological tests. Usually, however, the mean radial velocity of a cluster is estimated from redshift measures of only two or three individual members - too few to provide much information about the internal kinematics of the cluster. For a few clusters and groups, however, there are enough data to allow a more or less meaningful estimate of the dispersion of velocities to be made. Data for several such clusters are given in table 6. Successive columns list the name of the cluster or the Abell catalog number (or both), the mean radial velocity (corrected for galactic rotation), the square root of the dispersion in radial velocities, the number of galaxies whose measured redshifts were used in calculating these quantities, and the authority for the observations.

Table 6. Velocity data for several clusters

Cluster	$\langle V_r \rangle$ (km s^{-1})	$\langle v_r^2 \rangle^{1/2}$ (km s^{-1})	n	References
Leo group				
(around NGC 3627)	787	260	18	1, 2
Virgo cluster	1136	643	73	1
Elliptical component	950	550	33	3
Spiral component	1450	750	19	3
Fornax cluster				
($\alpha \approx 3½^h$; $\delta \approx$ -36°)	1452	287	12	1, 2
Pegasus I cluster				
(around NGC 7619)	3836	260	6	1
Group around NGC 383	5274	504	9	1
Cluster 194				
(around NGC 541)	5321	406	41	4
Cluster 426				
(Perseus)	5437	713	7	1, 2
Cluster 1656				
(Coma)	6866	932	46	5
Cluster 2199				
(around NGC 6166)	8736	541	15	6
Cluster 2151				
(Hercules)	10775	631	15	7
Cluster 1377	15269	358	4	1
Cluster 2065				
(Corona Borealis)	21651	1210	8	1

REFERENCES. - (1) Humason, Mayall, and Sandage 1956; (2) Mayall and de Vaucouleurs 1962; (3) de Vaucouleurs 1961b; (4) Zwicky and Humason 1964a (they do not state whether or not their radial velocities are corrected for galactic rotation); (5) Lovasich et al. 1961; (6) Minkowski 1961; (7) Burbidge and Burbidge 1959b.

Three points should be noted: (1) The square roots of the dispersion given are in *radial* velocities (i.e., as seen from the galactic center). If the velocity field throughout a cluster is isotropic, the true value should be $\sqrt{3}$ times the value given. If, on the other hand, the motions of member galaxies are largely radial in a cluster, the true square root of the dispersion should be less than $\sqrt{3}$ times the tabulated value because most of the galaxies measured are near the projected center of a cluster where a majority would be expected to be moving nearly in the line of sight. (2) Redshifts can be measured only for the brightest galaxies in most clusters. If a degree of statistical equilibrium exists in a cluster, the fainter galaxies should have a larger dispersion in velocity; if not, the tabulated values may be near the true ones, except for projection effects. (3) Typical individual measures of redshifts of galaxies carry probable errors of from 50 to 200 km s^{-1}. When only a few galaxies in a cluster are measured, the velocity dispersion can be considerably in error for this observational reason alone.

For a more current and complete compilation, see Noonan (1973).

4. DYNAMICS OF CLUSTERS

4.1. General Considerations

The incomplete, and even conflicting, observational data do not permit us to draw a very complete picture of the dynamics of even a single cluster. At best, from observational data alone, we can make only rough estimates of clusters' probable dynamical structures.

The time of relaxation in the central regions of a typical regular cluster, like the Coma cluster, is almost certainly greater than 10^9 years, and probably lies within a small factor of 10^{10} years (Oort 1958; van Albada 1960). It is doubtful, therefore, that in the presumed age of the Universe (~ 10^{10} years) gravitational encounters between galaxies can have set up anything approaching a state of statistical equilibrium in a regular cluster. If, however, such a state *could* have been reached, we would expect: (1) an equipartition of energy with a consequent segregation of velocities of galaxies according to their masses; (2) a radial segregation of galaxies within the cluster according to mass, with the less massive, faster-moving galaxies occupying a larger spatial distribution; and (3) a mass distribution within at least the central regions of the cluster resembling that of the isothermal polytrope. Observational verification of all of these effects has been reported by various investigators, but these observations are inconclusive.

In cluster 194, Zwicky and Humason (1964a) report radial velocities for 41 galaxies that they presume to be cluster members. The mean velocity is 5321 km s^{-1}, with an rms dispersion of 406 km s^{-1}. The brightest 21 galaxies, however, have a mean velocity of 5254 km s^{-1}, with a dispersion of 360 km s^{-1}, while the corresponding figures for the faintest 21 galaxies are 5392 and 439 km s^{-1}. Zwicky and Humason interpret the slightly higher dispersion in velocity of the fainter members to be indicative of a degree of equipartition of energy. The brightest six galaxies do have velocities very near the mean for the cluster, but for the remaining galaxies the effect vanishes; if anything, the dispersion in velocity *decreases* with increasing magnitude, as is seen at once on a plot of Zwicky and Humason's data. Indeed, as those authors themselves point out, there is a larger difference between the velocity dispersiont as was pointed out in the last section, this result depends on what assumption is made about the field. The writer's own investigation indicates that a slight segregation possibly exists (fig. 6), but not necessarily so. Rudnicki's (1963) finding of a segregation of bright and faint galaxies in cluster 426 may result from irregular interstellar absorption. The case for the Virgo cluster is unclear because it may be more than one cluster. The slight segregation found by Hodge, Pyper, and Webb (1965) in the Fornax cluster appears to be real, but Fornax is a poor cluster, and the segregation is at best small. Other investigations have also suggested slight radial segregation of bright and faint galaxies in some clusters (N. Bahcall 1972; Kwast 1966; Rudnicki and Baranowska 1966b; Rood 1969; Rood and Abell 1973; Noonan 1971), but the evidence is sometimes contradictory, and in no case is there segregation present in the amount required by complete statistical equilibrium.

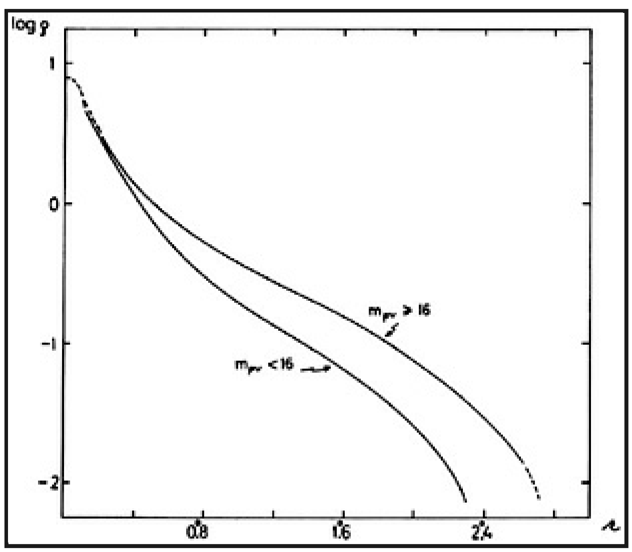

Figure 6. The radial density distribution of bright and faint galaxies in the Coma cluster (No. 1656) derived from Abell's counts. The zero points of the ordinates are arbitrary and have been shifted to agree for bright and faint galaxies at the cluster center.

As we have already seen, both Zwicky and N. Bahcall have shown that observations of the density distributions in several regular clusters are compatible with isothermal distributions, but their data can also be represented about as well by various other distributions that have nothing to do with statistical equilibrium.

In summary, there is no convincing evidence that equipartition of energy exists in any cluster, although there is some evidence for partial radial segregation by mass in a few clusters. Nor would we expect equipartition of energy, because of the long times of relaxation in clusters. Ages of clusters are probably at most only a few relaxation times; some dynamical evolution has probably taken place, but it is unlikely that a final state has been reached.

4.2. Masses of Clusters

If Newton's laws are valid over intracluster distances, then the virial theorem,

$$\frac{1}{2}\ddot{I} = 2T + \Omega, \tag{8}$$

should apply. If, further, a cluster is in a steady state, the polar moment of inertia should be constant, so $\ddot{I}$ = 0, in which case

$$2T + \Omega = 0. \tag{9}$$

Masses computed for clusters of galaxies are usually based on equation (9). The requirement that $\ddot{I}$ strictly vanish does not have to be rigorously fulfilled; if only the cluster is on the verge of stability, so that the total energy is zero, we should have

$$E = T + \Omega = 0, \tag{10}$$

and the mass derived from equation 10 is only a factor of 2 smaller than that obtained from equation (9).

The kinetic energy T can be approximated by $1/2\mathcal{M}_{\rm cl} <V^2>$, where $\mathcal{M}_{\rm cl}$ is the total cluster mass and $<V^2>$ is the velocity dispersion weighted by mass. Radial velocities are usually observed for the brighter and presumably the relatively massive cluster members; thus little error results from estimating $<V^2>$ from the observed dispersion in radial velocities in a cluster, $<V_r^2>$. The principal uncertainty is that of projection effects. If the velocity field is isotropic at all points in a cluster, then $<V^2>$ = $<3V_r^2>$; if most galaxies move radially through the cluster, then $<V^2>$ may only slightly exceed $<V_r^2>$.

The potential energy, Ω, is

$$- G \sum_{i \neq j} \frac{m_i m_j}{r_{ij}},$$

where the summation must be carried out over all pairs of galaxies of masses m_i and m_j separated by distance r_{ij}. If the cluster has n members, there are $n(n - 1) / 2 \approx n^2/2$ such pairs. It is customary to write the potential as $- G \mathcal{M}_{\rm cl}^2 / 2R'$, where $1/R'$ is a weighted mean of the $(1 / r_{ij})$'s. Attempts have been made to evaluate $1 / R'$ directly for a few groups, and even for the Hercules cluster (A2151) by the Burbidges (1959b). More often, R' is associated with the apparent radius of a cluster. For rich symmetrical regular clusters, the potential can be expressed more conveniently:

$$- \Omega = G \int_0^R \frac{\mathcal{M}(r) d\mathcal{M}(r)}{r}, \tag{11}$$

where $\mathcal{M}(r)$ is the mass contained within a radial distance r of the cluster center. For a constant mass-to-light ratio, we can write $\mathcal{M}(r) = fL(r)$, which leads to

$$- \Omega = G f^2 \int_0^R \frac{L(r) dL(r)}{r}. \tag{12}$$

The integral in equation (12) can be evaluated numerically from the density distribution of luminosity in the cluster derived from observations, leading finally to

$$- \Omega = qG\mathcal{M}_{\rm cl}^2/R, \tag{13}$$

where R is the outer cluster radius and q is a numerical factor that is near unity. For the Coma cluster, for example, Rood (1965), using the writer's observations, finds that q = 0.92. The only unknown in equation (9) (or eq. [10]) is thus the total cluster mass. Setting q = 1, we find

$$\mathcal{M}_{\rm cl} = \xi \langle V_{\rm r}^2 \rangle R/G, \qquad (14)$$

where the factor ξ allows for the factor of 2 uncertainty in whether equation (9) or equation (10) is more appropriate, and for the projection effects between $\langle V_r^2 \rangle$ and $\langle V^2 \rangle$; ξ thus lies in the range 1/2 to 3.

Although the dynamical method of estimating masses of galaxian clusters is well known, the assumptions involved have been reviewed here because the masses found often seem discordant with those estimated from the luminosities of the cluster members. Ratios of mass to light (in solar units) found for individual galaxies (e.g., Burbidge and Burbidge 1959a) or for double galaxies (Page 1962) range from 1 to 15 for spirals and from 10 to 70 for ellipticals and S0's. On the other hand, to reconcile the masses estimated for a number of groups and clusters of galaxies from dynamical and luminosity methods, mass-to-light ratios for some of those systems would have to lie in the range from 100 to 1000 (e.g., Limber 1962). Years ago, Zwicky (1933), and Smith (1936) called attention to the unexpectedly high mass obtained for the Virgo cluster from its internal kinematics. The Burbidges (1959b) have called attention to the same situation in the Hercules cluster (A 2151). Other discussions of the problem of the high dynamical masses found for groups and clusters include those of de Vaucouleurs (1960), Burbidge and Burbidge (1961), van den Bergh (1961c), Limber (1961), and Rood (1965). The problem, then, is to understand why mass-to-light ratios found for clusters and groups of galaxies are often higher - by even a factor of 10 or more - than those found for individual or double galaxies. Among the possibilities proposed are the following: 1. Ambartsumian (1958) has suggested that clusters (at least some of the smaller groups) have positive energy and are expanding. On the other hand, to reduce the dynamical mass of a cluster even by just a factor of 4 from that given by the virial theorem, we would need to have $T = -2\Omega$; in this case, the observed speeds of galaxies are, in the mean, only $\sqrt{2}$ times greater than the speeds they will have when the cluster has expanded to infinity. In a period of at most a few times 10^9 years, all clusters would dissipate enough to have lost their identity, and it would be difficult to understand why most galaxies are still in clusters.

2. Perhaps the high masses of some clusters are due to, a few extremely massive galaxies. There does appear to be a tendency for the mass-to-light ratio to be an increasing function of mass for elliptical galaxies (Rood 1965). However, it is very hard to invent convincing stellar populations that can give the extremely high mass-to-light ratios that would be required. Moreover, many of the smaller clusters and groups for which the mass-to-light ratio appears to be high have only spirals among their massive members.

3. The derived mass-to-light ratios are proportional to the assumed value of the Hubble constant. A greatly expanded distance scale would therefore reduce them. But it would also reduce the corresponding ratios found for most individual galaxies and for double galaxies, so the discrepancy would remain.

4. There is a probability that metagalactic structure of larger size than clusters exists i.e., second-order clusters (see next section). If so, and if gravitational forces exist between clusters in such a system, then two or more clusters seen overlapping in projection may occasionally be mistaken for an isolated system, and their relative motions can lead to a spuriously high observed velocity dispersion. In some cases, such an effect may partially be responsible for the mass-to-light discrepancy. The Hercules cluster, in particular, is in a region rich in groups and clusters of about the same distance and may not be a single dynamic unit. De Vaucouleurs's suggestion that the Virgo cluster is at least two clusters has already been mentioned, as has the possibility of inappropriately assuming cluster membership for outlying galaxies in the Coma cluster. In fact, Gott, Wrixon, and Wannier (1973) have shown that some systems that have been identified as groups may actually be optical alignments of field galaxies that are not gravitationally bound at all, and thus that high mass-to-light ratios reported for these systems may be entirely spurious.

5. Subclustering, or incidence of duplicity within a cluster, results in lowering the average separation of galaxies, thereby increasing the potential energy for a given mass. Van den Bergh (1960) has pointed out that binary galaxies in the Virgo cluster can possibly affect significantly the derived cluster mass. Studies of subclustering in several clusters, however (Abell, Neyman, and Scott 1964), show that the phenomenon can not be important generally.

6. A popular explanation is that clusters contain a large amount of invisible matter which adds to their masses but not to their visual luminosities. Zwicky (1957) believes that intergalactic obscuring matter in clusters hides more remote clusters, although the existence of such absorption is not universally accepted. For the Coma cluster, he estimates an obscuration of 0.6 mag (Zwicky 1959). The dust required to produce this absorption, however, corresponds to a density of only 10^{-30} g cm^{-3}, if the dust has the same obscuring properties as interstellar grains in the Galaxy (de Vaucouleurs 1960); this would be a negligible contribution to the cluster mass. Evidence for luminous haze in the centers of clusters is inconclusive and controversial; if *visible* luminous matter does exist, it can in no case contribute substantially to the mass of a cluster. Field (1959) has shown that neutral hydrogen cannot be important enough in intergalactic space to affect cluster masses measurably. Observations of the 21-cm line in absorption reported by Robinson, van Damme, and Koehler (1963) suggest the presence of neutral hydrogen in the Virgo

cluster, but the total mass indicated is more than two orders of magnitude less than that of the cluster itself. Recent 21-cm observations of three groups by Gott, Wrixon, and Wannier (1973) indicate that neutral hydrogen falls short by at least a factor of 25 of having enough mass to gravitationally bind those systems. Nor is hot plasma likely to be able to contribute appreciably to the potential energies of clusters; an analysis by Davidson, Bowyer, and Welch (1973) of the radio, soft X-ray, and far-ultraviolet observations of the Coma cluster, for example, seems to rule out the possibility that ionized gas contributes enough mass to bind that cluster. Still, intracluster material cannot yet be ruled out in all cases.

Actually, the discrepancy may not be serious for rich clusters. Luminosities, masses, and mass-to-light ratios for six relatively rich to rich clusters are computed from the data in tables 2 and 6 with equation (14), and are listed in table 7. The factor ξ has been set equal to 2.1, following Rood's (1970) analysis of the Coma cluster velocities. The uncertainty in ξ can affect a derived cluster mass by at most a factor of about 2. If, for example, the clusters have zero energy, so that equation (10) rather than (9) applies, the actual mass-to-light ratios would be half those given in table 7. Total radii of 1.2 and 4.0 Mpc are used for the Virgo and Coma clusters, respectively. The Corona Borealis cluster was also assumed to have a radius of 4.0 Mpc, and 3.0 Mpc was arbitrarily chosen for the other clusters, for which the writer has not yet determined radii observationally. A Hubble constant of 50 km s^{-1} Mpc^{-1} is adopted throughout. The visual luminosity is uncertain for cluster 1377, for which only the old photometry of Baade is available, but it is probably correct to within a factor of 2; the velocity dispersion for this cluster, however, is calculated from only four radial velocities, so the mass is very uncertain. For cluster 194, the luminosity was calculated from magnitudes published by Zwicky and Humason (1964a; Abell 1964). Zwicky and Humason (1964b) later reported that some (they do not say which) of the galaxies whose magnitudes they published were subsequently discovered not to be cluster members, but it is doubtful that elimination of those objects can affect the luminosity given in table 7 by as much as a factor of 2. The $\mathcal{M} / L$ ratio of 144 found for the Coma cluster can be compared to the value of about 165 found by Rood et al. (1972) (adjusted to the Hubble constant used here).

Table 7. Mass-light ratios for several rich clusters

Cluster	Mass $\mathcal{M}$ ξ = 2.1 ($\mathcal{M}_{\odot}$)	Visual Luminosity, L (solar units)	$\mathcal{M} / L$
Virgo	2.4×10^{14}	1.3×10^{12}	181
194	2.4×10^{14}	3.8×10^{12}	64
1377	1.9×10^{14}	2.7×10^{12}	70
1656(Coma)	1.7×10^{15}	1.2×10^{13}	144
2065 (Corona Borealis)	2.9×10^{15}	1.2×10^{13}	231
2199 (around NGC 6166)	4.3×10^{14}	6.1×10^{12}	71

The clusters listed in table 7 are the only rich ones for which the writer has been able to find enough data to compute mass-to-light ratios. The ratios listed are all of the order 10^2 or less. For elliptical galaxies in binary systems, Page (1962) finds $\mathcal{M}/L \approx 50$ (for H = 50 km s^{-1} Mpc^{-1}). There may still be a discrepancy of a factor up to 3 because of the uncertainty of ξ, but this reexamination of the data shows that a *serious* mass discrepancy does not *necessarily* exist in rich clusters.

4.3. Formation and Evolution of Clusters

It is highly improbable that clusters of galaxies have been built up by chance encounters of galaxies in the general field. Close encounters of three or more bodies of comparable mass can lead to the formation of stable pairs. However, the time required to build a cluster is very great; even if the Universe were old enough (> 10^{12} years), most galaxies would still be in the general field rather than in clusters, as the observations suggest. Moreover, if the galaxies in a three-body encounter do not have comparable mass, the energy exchange is too small to produce captures, and observed clusters containing galaxies whose masses range through several orders of magnitude could never be built up by such a process (Ambartsumian 1961). We are nearly forced to conclude, therefore, either that clusters are systems whose member galaxies became gravitationally bound at more or less the same time, or that the clusters represent condensations from pregalaxian material and that subcondensations within them became galaxies.

Many investigators have attempted to determine conditions under which galaxies or clusters can condense from gas or plasma. As yet, there is no successful complete theory for the formation of galaxies and clusters from primordial material. On the other hand, a theory of van Albada (1960, 1961, 1962) gives a rather detailed account of how clusters may form and evolve, given certain initial conditions. His theory is of sufficient interest to warrant a brief review here.

Van Albada assumes that galaxies already existed in a nearly homogeneous expanding universe before the clusters formed. He then considers the conditions under which a region of the expanding universe can become gravitationally unstable. For the sake of mathematical amenability, he considers only spherically symmetrical condensations. For the case of zero cosmological constant, he finds that density fluctuations of only about 2 to 7 percent can lead to gravitational instabilities, which result in the formation of a cluster containing matter originally spread over a region of radius

$$r \approx \sigma / (G\rho)^{1/2}, \quad (15)$$

where ρ is the smoothed-out density of the universe at the time the instability commences, and σ is the square root of the velocity dispersion. For the latter, van Albada adopts 50 km s^{-1}. If the radius of the universe is R, ρ varies as R^{-3} and σ as R^{-1}; thus r varies as $R^{-1/2}$. The instabilities, therefore, occur over regions that are relatively smaller as the universe expands. At an early history of the expansion, when R is small, one would expect condensations which would lead to large-scale inhomogeneities, contrary to the cosmological principle. [(3)] Instabilities must therefore be inhibited somehow at very early epochs, until ρ has decreased to a value of 10^{-24} or 10^{-25} gm cm^{-3}; van Albada suggests that radiation pressure might provide this inhibiting force.

With numerical calculations, van Albada follows the growth of an instability and its subsequent evolution. Since the galaxies already exist when the condensation begins, there is no dissipation of kinetic energy by cooling. A central nucleus develops and steadily increases in density and velocity dispersion. Matter streams into the nucleus from a surrounding corona, which in turn attracts matter from the entire unstable region. As the nucleus contracts, the corona changes only slightly. Meanwhile, the expansion of the universe makes the cluster appear relatively more and more prominent against the surrounding field. The density distribution in the inner regions of the cluster is compatible with those observed - for example, with an isothermal polytrope. At no time, however, is the cluster in statistical equilibrium. The condensations develop most slowly from the initial density perturbations of the smallest amplitude. It is possible therefore, that an early instability over a large relative region containing a large mass could begin with a small-amplitude density excess over the mean, and just now be developing into a great cluster. A model of such a system with an age of several billion years has dimensions and structure roughly resembling the Coma cluster.

A point to be remembered is that in the van Albada picture the galaxies exist before clusters form; it is large-scale instabilities in a "fluid" of point-mass galaxies that produce the clusters. Moreover, at early stages the contraction of a cluster is only "contraction" relative to the expanding universe. At later stages the nucleus, as it increases in mass, may contract in an absolute sense, but the increasing velocity dispersion eventually inhibits further contraction and the nucleus expands again. Because the outer layers of the cluster are always expanding absolutely, they cannot stop the expansion of the nucleus, and van Albada expects the cluster ultimately to dissipate. None of his detailed models, however (at the time of writing), have been carried to such an advanced state; in fact, none of them may even correspond to development as advanced as that of actual clusters.

A different approach to the study of the dynamical evolution of clusters is that of Aarseth (1963, 1966, 1969), who begins with a cluster of n members in an arbitrary configuration and with arbitrary initial velocities, and simply solves numerically the "n-body problem." He has considered clusters of from 25 to 100 members, interacting under purely gravitational forces. To take account of the finite sizes of galaxies, Aarseth chooses for the potential function of a galaxy,

$$\Phi = -\frac{G\mathcal{M}}{(r^2 + \xi^2)^{1/2}}, \quad (16)$$

where ξ is its effective radius.

One of Aarseth's most interesting models, and one which may provide some approximation to an actual cluster of galaxies, has 100 members, distributed initially with a spherically symmetrical density configuration of the form

$$\rho(r) = \rho(0)[1 - r/R]. \quad (17)$$

The 100 objects (representing galaxies) have four different masses: 40 have a relative mass of 0.25; 30 of 0.75, 20 of 1.50, and 10 of 3.75. The complete mass range is thus over a ratio of 15-1. All members are started with random velocities, with the restriction that no object has an initial velocity that will carry it to a greater radial distance from the cluster center than R. In particular, there is no initial correlation between mass and velocity. Aarseth's integration follows the cluster for a period which would correspond in a real galaxian cluster to about 1010 years. During that time, he finds that a nucleus develops which is surrounded by an extended halo. A few of the halo members escape. At the end of the integration period there is a slight radial segregation of members by mass, but the least massive objects have a radial distribution less than 40 percent greater than the most massive ones, and the extreme range in mean kinetic energy, originally over a factor of 15, is reduced by only about half; in other words, the cluster is far from a state of statistical equilibrium. Application of the virial theorem in a naïve way to find the mass of the cluster, however, would lead to a mass error of less than 20 percent. Aarseth finds that about 50 collisions would have occurred in the cluster over the 10^{10} years. Only a few stable binary systems are formed, but those have long lifetimes. A similar model cluster with 50 members showed qualitatively similar evolution.

Aarseth also followed the evolution of two irregular model clusters of 50 members each. One was initially V-shaped with its members at rest, and the other had its members spread evenly over an elliptical region, and with small random velocities. In each case a stable "subsystem" formed as a cluster nucleus that retained stability over the entire integration time. Even for these clusters, the mass found from application of the virial theorem would be correct to within 50 percent.

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[3] The correctness of the cosmological principle has never been observationally verified more than very roughly. Back.

5. THE DISTRIBUTION OF CLUSTERS

5.1. The Evidence for the Local Supercluster

A local irregularity in the distribution of nearby groups and clusters of galaxies has been suspected for several decades. From his analysis of the distribution of double and multiple galaxies, Holmberg (1937) inferred the existence of a metagalactic cloud with a diameter, according to the distance scale used in this review, between 90 and 150 Mpc, and with a center lying in the general direction of the north galactic pole at a distance of about 18 Mpc. Holmberg's conclusion was confirmed qualitatively by Reiz's (1941) study of the distribution in direction and magnitude of 4000 galaxies in the northern galactic polar cap. The idea of a local supercluster of galaxies was revived by de Vaucouleurs (1953, 1956, 1958), who described it in considerable detail. According to de Vaucouleurs, the supercluster has a diameter (with H = 50 km s^{-1} Mpc^{-1}) of about 75 Mpc, and it contains, in addition to the Local Group, the Virgo cluster, the Ursa Major cloud, and numerous smaller groups and clusters. He finds the system to be flattened, so that the bright galaxies are seen in the sky highly concentrated toward a great circle (the "supergalactic equator") with its pole at $l^{II} = 47°$, $b^{II} = 5°$. He believes the center of the system to lie within or near the Virgo cluster. The flattening suggests rotation; from his analysis of the radial velocities of bright galaxies, de Vaucouleurs presents interesting evidence for differential rotation, and derives about 500 km s^{-1} for the rotational velocity of the Galaxy about the center of the system. From this rotational velocity, he derives a total mass for the Local Supercluster of the order of 10^{15} $\mathcal{M}_{\odot}$. On the other hand, the present or "instantaneous" period of revolution of the Galaxy is about 2×10^{11} years; and even though de Vaucouleurs believes the system to be expanding slowly, it can hardly have completed even one rotation unless it was formed at a very early epoch in the expansion of the Universe, when its mean density was orders of magnitude higher than at present. Quite possibly, therefore, the apparent flattening of the supercluster may have nothing to do with its presumed rotation.

The dynamical properties of the Local Supercluster may not be well established, but further evidence for its reality as a geometrical entity is provided by an independent investigation by Carpenter. Carpenter (1961; Abell 1961) studied the distribution in magnitude and direction of galaxies brighter than m_{pg} = 16 in a large region of the north galactic hemisphere from the Palomar Sky Survey prints. At magnitudes brighter than 13.5, he finds a highly significant concentration of galaxies along a 90° sector of an 18° strip along de Vaucouleurs's "supergalactic equator." In the next interval of 1 mag, the number of galaxies drops off very rapidly compared with expectations for a uniform galaxy distribution in depth. For $m_{pg} > 14.5$, however, the logarithm of the number of galaxies brighter than m_{pg} increases as $0.6m_{pg}$, which would be expected if most of these galaxies are remote ones, beyond the limits of the supercluster. Carpenter finds a similar result for galaxies in adjacent 18° strips saddling the strip along the supergalactic equator, except that the total number of bright galaxies in these zones is less than in the central strip.

5.2. Other Evidence of Second-Order Clusters

The tendency of clouds, clusters, and groups of galaxies to form assemblages of higher order than single clusters was noted long ago by Shapley (1933, 1957). The phenomenon of superclustering was demonstrated dramatically, however, by thhese systems are described by Dr. Shane elsewhere in this volume. Typical dimensions of these clouds (for H = 50 km s^{-1} Mpc^{-1}) lie between 15 and 60 Mpc.

Even the very rich clusters in the writer's catalog (Abell 1958) show a strong tendency for second-order clustering, Of the 2712 clusters catalogued, 1682 were selected as comprising a homogeneous statistical sample. Clusters in the sample all have populations (defined in Section 3.4) of at least 50, redshifts in the range $d\lambda / \lambda$ = 0.02 - 0.20, and lie at great enough galactic latitudes that interstellar absorption does not prevent their identification (usually at latitudes greater than about 30°). The surface distribution of these clusters is shown in figure 7. The clusters are classified according to distance, the mean redshifts of clusters in distance groups 1 through 6 being, respectively, 0.027, 0.038, 0.067, 0.090, 0.140, and 0.180.

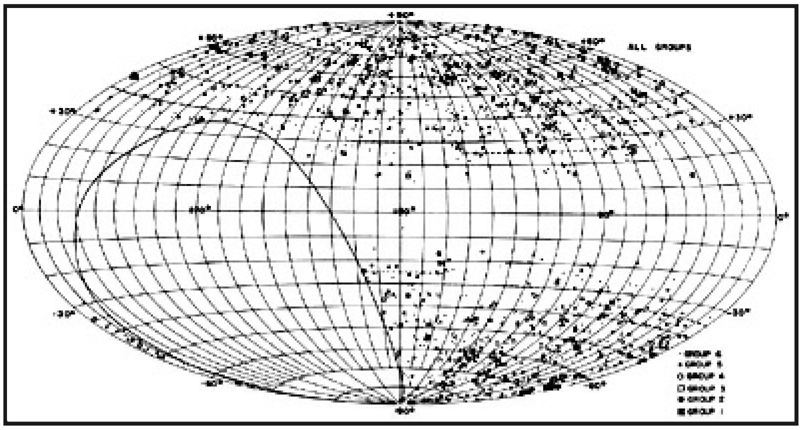

Figure 7. The distribution in galactic coordinates (old system) of the clusters in Abell's catalog. Those clusters closer to the galactic equator than the dotted line are not part of the homogeneous statistical sample. The large empty oval region is the part of the sky not covered by the Palomar *Sky Survey*. The plot is on an Aitoff equal-area projection.

Superficial examination of figure 7 shows an obvious clustering tendency of the clusters themselves. To test the significance of possible superclustering, the part of the sky covered by the homogeneous sample was divided into grid cells of various sizes, and for each sized cell the frequency distribution, $f(t)$, of cells containing t clusters each was determined. A χ^2 test was used to estimate the probability that $f(t)$ would be obtained in a random sampling from a population whose frequency distribution is the binomial distribution, $B(t)$. The $f(t)$ and comparison with $B(t)$ was determined separately for clusters in distance groups 5 and 6, and for clusters in distance groups 1 through 4 combined. It was found that $f(t)$ approaches $B(t)$ for very small cell sizes, for then every cell contains either one cluster or none. With increasing cell size, $N(t)$ departs more and more from $B(t)$; the probability $P(\chi^2)$, of $N(t)$ being a random sampling from a population with frequency distribution $B(t)$ for the most distant clusters (for which the sample is largest), is as low as 10^{-30} to 10^{-40}. For larger cell areas $P(\chi^2)$ increases again, mainly because the sample size diml area ones) and the deviation of $N(t)$ from $B(t)$ is less significant. $P(\chi^2)$ should also eventually increase with cell size if the cells become large compared to any anisotropics in the cluster distribution - that is, if superclustering is "smoothed out." The writer originally interpreted an observed inverse correlation between the cell diameters for which $P(\chi^2)$ is a minimum with the cluster distance class as an indication that the second-order clustering occurs on the same scale at all distances surveyed. This interpretation is not strictly justified because of the smaller significance of the results for large cell sizes. However, at cell sizes smaller than those for which $P(\chi^2)$ is at a minimum, the descent of $P(\chi^2)$ with cell size is steepest for the most distant clusters and least steep for the nearest, as one would expect for superclusters of a common scale displaying smaller angular sizes at greater distances.

The evidence that second-order clusters may have similar linear sizes at different distances argues against their being illusions produced by interstellar or intergalactic obscuration. Simple inspection of figure 7 would also seem to rule out absorption as the cause of the clumpy cluster distribution; if apparent clumps of relatively near clusters are merely parts of a uniform or random distribution of clusters seen through holes in absorbing material, then apparent clumps of more remote clusters should be seen in the same directions, but certainly not between them, as is the case.

About 50 apparent groupings of clusters - probably second-order clusters - can be identified in figure 7. The writer has described 17 of these groupings (Abell 1961). The mean number of clusters (in the homogeneous sample) among the 17 second-order groups is 10.6±6.0 (s.d.). This number, of course, refers only to the very rich clusters in the Abell catalog; the total number of clusters and groups of all kinds in a typical second-order cluster might be greater by an order of magnitude or more. The mean linear diameter of the 17 groups of clusters is 78.0± 23.8 (s.d.) Mpc. The list of 17 systems includes two of the Shane clouds - the Corona cloud and the Serpens-Virgo cloud - described elsewhere in this volume.

The distribution of clusters in Volumes 1, 2, 3, and 5 of the *Catalogue of Galaxies and Clusters of Galaxies* (Zwicky et al.

1961-1968), the first four volumes of the catalog to be published, have been analyzed in the same way as were those in the Abell catalog with nearly identical results by Abell and Seligman (1965, 1967).

Numerous other investigators have attempted analyses of the distribution of rich clusters of galaxies in the published catalogs. Among them, Kiang and Saslaw (1969) computed serial correlations of Abell clusters in 50-Mpc cubic cells to determine the three-dimensional cluster distribution, and find correlations over a scale of at least 100 Mpc and possibly to 200 Mpc. Bogart and Wagoner (1973) performed nearest-neighbor tests on the Abell clusters, and found that the distribution of nearest-neighbor distances from half of the clusters (sources) to the other half (objects) has a significantly smaller mean than does the corresponding distribution when a set of random points is used for sources, indicating that the clusters are significantly clustered. Bogart and Wagoner estimated the scale of the clustering by rotating the "object" half of the clusters in galactic longitude until the distribution of nearest-neighbor distances approached the random one. The angular scale found for distance group 5 clusters is slightly greater than for the more distant group 6 clusters, suggesting a physical association of clusters; the corresponding linear scale is ~ 200 Mpc.

Statistical analyses of three catalogs of extragalactic objects have been carried out recently by the Princeton group (Peebles 1973; Hauser and Peebles 1973; Peebles and Hauser 1974; Peebles 1974). The sources are the Abell (1958) catalog, the galaxies catalogued by Zwicky and his associates (Zwicky et al. 1961-68), and the galaxies catalogued from the Lick Astrographic plates (Shane and Wirtanen 1967). Peebles and Hauser have investigated the correlation between objects in the individual catalogs and the cross correlations of objects in different catalogs. They find that the clusters and individual galaxies seem to correlate in direction, both separately and with each other over angular distances of up to 6°. The linear size of these homogeneities is of the order 100 Mpc.

Peebles has also developed a powerful statistical method for detecting variations over the surface distribution of galaxies or clusters by means of a two-dimensional power spectrum. It was first applied (Yu and Peebles 1969) to test the hypothesis of complete second-order clustering of the Abell catalog clusters. Yu and Peebles found that if second-order clusters contain an average of 10 rich clusters each, then only about 10 percent or less of the Abell clusters can be members of such superclusters, and that in a model of complete superclustering, on the average there could be at most about 2 clusters per supercluster. It should be noted that in these calculations those clusters of distance class 5 in the southern galactic hemisphere, where inspection of figure 7 suggests second-order clustering to be most pronounced, were omitted because that part of the Abell catalog seemed to Yu and Peebles to be atypical.

Peebles (1973) developed the power-spectrum approach further, and he and Hauser reanalyzed the Abell catalog (Hauser and Peebles 1973). They report "clear and direct evidence of superclusters with small angular scale" and that the structure corresponds to an average of 2 to 3 clusters per supercluster.

The early χ^2 tests described above are subject to misinterpretation because of the possibility of a general absorption gradient and other systematic effects, and the results of these tests alone should thus be viewed with caution. However, as we have seen, the same results are obtained with more sophisticated tests, made possible with modern computing equipment, especially the powerful power-spectrum analysis. These studies of the catalogs of observed galaxies and clusters of galaxies show very strong - perhaps overwhelming - evidence for inhomogeneities in the large-scale distribution of matter in space with a scale (for H = 50 km s^{-1} Mpc^{-1}) of the order of 10^8 pc.

If Newton's laws are valid over dimensions of second-order clusters, and if the latter do not partake of the general expansion of the Universe, we can use the virial theorem to estimate the velocity dispersion within such a system. We denote the mean separation of its members by R', and have

$$\langle V^2\rangle^{1/2} = 4.6 \times 10^{-2}(\mathcal{M}/R')^{1/2} \text{ km s}^{-1}, \tag{18}$$

where $\mathcal{M}$ and R' are in solar masses and parsecs, respectively. If the mass of a typical rich cluster is $5 \times 10^{14}\ \mathcal{M}_\odot$, the entire mass of a typical supercluster probably lies in the range 10^{15} to $10^{17}\ \mathcal{M}_\odot$. Adopting 20 Mpc for R', we find that $\langle V^2\rangle^{1/2}$ should lie in the range 300-3000 km s^{-1}. If the velocity field is isotropic, the observed rms dispersion in radial velocity should actually lie under 2000 km s^{-1}.

Radial velocities are known for six clusters that are suspected of forming a second-order cluster covering an elongated region centered near $\alpha = 16^h 14^m$, δ = + 29° (Abell 1961). The total range of these six velocities is about 3000 km s^{-1}. There are not enough data to determine a meaningful velocity dispersion for the system, but at least the observations are not incompatible with the assumption that gravitational interactions occur between its members.

If second-order clusters are expanding, or if they do not have negative total energy, the observed dispersion in radial velocities could be higher than the value derived above. Suppose, for example, that gravitational forces within such a system are negligible, and that it expands at the general universal rate. Then the corresponding spread of radial velocities across a second-order cluster of diameter D Mpc should be $\Delta V \sim H \times D = 50 \times 75 = 3750$ km s^{-1}. Since our estimate of the value of D is proportional to H^{-1}, the derived value of ΔV is independent of the value assumed for H.

5.3. The Large-Scale Distribution of Clusters and the Mean Density of Matter in the Universe

Hubble's (1934) study of the distribution of galaxies revealed by 60- and 100-inch telescope photographs showed that to the accuracy of the observations the galaxies, except for a clustering tendency, appeared to be distributed uniformly in depth in all directions. The distribution of the rich clusters has similarly been investigated by the writer (Abell 1958). The homogeneous sample selected from the Abell catalog yields the frequency distribution $N(z)$, of clusters with (estimated) redshift, z. There is, as we have seen, a small-scale clustering tendency of the clusters themselves, and $N(z)$ cannot be determined (from these data) accurately enough to choose between different cosmological models. There is, however, no evidence for departure from uniformity over regions of space that are large compared with the scale of the second-order clustering. Neither the Hubble nor the Abell data are accurate or complete enough to ensure that the cosmological principle is precisely realized throughout the observable region of space, but both are at least compatible with the assumption of large-scale homogeneity.

An important cosmological datum is the mean density of matter in space. The available velocity-dispersion data permit us to make meaningful estimates of the possible range of the density of the Universe - estimates which include any unseen matter within the clusters that can produce dynamical effects.

The mean density of a rich cluster gives us an upper limit. If the Corona Borealis cluster (number 2065) has a mass of 3×10^{15} $\mathcal{M}_{\odot}$ (probably an upper limit), it has a mean density of the order of 10^{-27} g cm^{-3}. Because cluster 2065 is an unusually rich and compact one, typical rich clusters probably have mean densities an order of magnitude lower. A more realistic upper limit is the mean density of a typical second-order cluster. For a mass of 10^{16} $\mathcal{M}_{\odot}$ and a radius of 40 Mpc, such a system would have a mean density of the order 10^{-30} g cm^{-3}.

A lower limit to the density of the Universe is found by assuming that all of its mass is contained in clusters as rich as those in the writer's catalog. After correcting for the fact that the statistical sample does not cover the entire sky, we estimate that about 4000 such clusters probably exist within a distance of 1.2×10^9 pc (corresponding to the distance of distance class 6 clusters). If 10^{14} $\mathcal{M}_{\odot}$ is a lower limit to the mass of a rich cluster, there are at least 8×10^{50} grams of matter within a distance of 1.2×10^9 pc, corresponding to a mean density of about 4×10^{-33}. This estimate is probably too low for the mean density of the Universe by at least an order of magnitude.

Thus, we estimate that the mean density of that matter in the Universe whose gravitational influence produces observable kinematical effects lies in the range of 10^{-32} to 10^{-30} g cm^{-3}. The best guess, to order of magnitude, is 10^{-31} g cm^{-3}. A Hubble constant, H = 50 km s^{-1} Mpc^{-1}, is assumed for these estimates; of course, the density estimate is proportional to H^2. The corresponding value of the deceleration parameter, q_0 (which is independent of H), is 10^{-2}.

Note added 1974 January 18. - The manuscript for this chapter was submitted early in 1966. Because of unavoidable delays in the publication of this volume, the chapter has become outdated in many respects. When the author received the manuscript with copy editing for final printing, he attempted to incorporate some recent references. Time did not permit a complete revision, however, and much of the chapter still has the flavor of an 8-year-old review. In particular, Sections 3.6 and 4.3 should be read with cognizance that much recent work is not reflected therein.

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