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PARTICLE PHYSICS IN THE EARLY UNIVERSE

Edward W. Kolb

Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

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1. INTRODUCTION

Perhaps the most striking illustration of the true unity of science is the development of the interdisciplinary field of ``particle cosmology." Particle physics examines nature on the smallest scales, while cosmology studies the universe on the largest scales. Although the two fields are separated by the scales of the objects they study, they are unified because it is impossible to understand the origin and evolution of large-scale structures in the universe without understanding the ``initial conditions" that led to the structures. The initial data was set in the very early universe when the fundamental particles and forces acted to produce the perturbations in the cosmic density field. A complete understanding of the present structure of the universe will also be impossible without accounting for the dark component in the density field. The most likely possibility is that this ubiquitous dark component is an elementary particle relic from the early universe.

The study of the structure of the present universe may reveal insights into events which occurred in the early universe, and hence, into the nature of the fundamental forces and particles at an energy scale far beyond the reach of terrestrial accelerators. Perhaps the early universe was the ultimate particle accelerator, and will provide the first glimpse of physics at the scale of Grand Unified Theories (GUTs), or even the Planck scale.

As a cosmologist I am interested in events that happened a long time ago. But in studying the past, I believe it is best to take the approach of a historian rather than an antiquarian. Now an antiquarian and a historian are both interested in things from the past. But an antiquarian is interested in old things just because they are old. To an antiquarian, there is no difference between a laundry list from June 1215 and the Magna Carta: they are both equally old. A historian, on the other hand, is interested in the past because it shapes the present. The job of a historian is to sort through events of the past and see which are important and which are not. I am not interested in the early universe just because it happened a long time ago, or it was really hot, or it was a bang (a really, really big one). The real reason I study the early universe is that events which occurred in the early universe left an imprint upon the present universe.

In these lectures I will concentrate on two events which occurred in the early universe. The first is the generation of perturbation in the density field during an early period of rapid expansion known as cosmic inflation. The second is the genesis of dark matter. The record of these events is written in the arrangement of galaxies, galaxy clusters, and imperfection in the isotropy of the cosmic microwave background radiation. If we really understood particle physics, we could predict the nature of those patterns. If we really knew how to read the story in the structures, we would learn something about particle physics. The story is there on the sky, patiently waiting for our wits to become sharp enough to read it.

In these lectures I will discuss the early universe. So the first thing we must do is to follow the procedure outlined by William Shakespeare [1]:

> Now entertain conjecture of a time
> When creeping murmur and the poring dark
> Fills the wide vessel of the universe.

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Cren Frayer cd@ipac.caltech.edu
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2. THE DENSITY FIELD OF THE UNIVERSE

The universe is not exactly homogeneous and isotropic, but it is a sufficiently accurate description of the universe on large scales that it is useful to consider homogeneity and isotropy as a first approximation, and discuss departures from this idealized smooth universe.

Let us begin by considering the density field, $\rho(\vec{x})$. If the average density of the universe is denoted as $<\rho>$, then we can define a dimensionless density contrast $\delta(\vec{x})$ as

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \langle\rho\rangle}{\langle\rho\rangle} . \qquad (1)$$

Of course we cannot predict $\delta(\vec{x})$, but we can hope to predict the statistical properties of $\delta(\vec{x})$. The correct arena to discuss the statistical properties of the density field is in Fourier space, where one decomposes the density contrast into its various Fourier modes $\delta_{(\vec{k})}$:

$$\delta(\vec{x}) = V \int d^3k \, \delta_{\vec{k}} \, e^{-i\vec{k}\cdot\vec{x}}, \qquad (2)$$

where *V* is some irrelevant normalization volume. After a little Fourier manipulation and some mild assumptions about the density field, it is easy to show that the two-point autocorrelation function of the density field can be expressed solely in terms of $|\delta(\vec{x})|^2$:

$$\langle\delta(\vec{x})\delta(\vec{x})\rangle = A \int_0^\infty \frac{dk}{k} k^3 \left|\delta_{\vec{k}}\right|^2 , \qquad (3)$$

where *A* is yet another irrelevant constant.

So long as the fluctuations are Gaussian, all statistical information is contained in a quantity known as the *power spectrum*, which can be defined as either

$$\begin{aligned} \Delta^2(k) &= k^3 \left|\delta_{\vec{k}}\right|^2 , \qquad \text{or} \\ P(k) &= \left|\delta_{\vec{k}}\right|^2 . \end{aligned} \qquad (4)$$

The first choice is much more physical, as it represents the power per logarithmic decade in the fluctuations. Although the first choice makes much more sense, the second choice is what is usually used. It turns out that graphs of $P(k)$ have a nicer form (but less physical content) than corresponding graphs of $\delta^2(k)$. Since the widespread availability of color graphics, presentation seems to be everything, and information content of secondary concern.

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2.1 The Power Spectrum From Large-Scale Structure

The power spectrum is related to the *rms* fluctuations in the density on scale $R = 2\pi / k$. The exact relationship depends upon sampling procedure, window functions, etc. But for a simple intuitive feel, imagine we have mass points spread throughout some sample volume. Now place a sphere of radius R in the volume and count the number of points within the sphere. Then repeat as often as you have the time or patience to do so. There will be an average number $\langle N \rangle$, and an *rms* fluctuation $\langle (\delta N / \langle N \rangle)^2 \rangle^{1/2}$. The power spectrum is related to that *rms* fluctuation: $\delta (k = 2\pi R^{-1}) \propto \langle (\delta N / \langle N \rangle)^2 \rangle^{1/2}$. Repeating the procedure for many values of R will give δ as a function of R - the power spectrum.

Now how does one go about observing the mass within a sphere of radius R? Well, it is difficult to measure the mass. It is easier to count the number of galaxies. So one assumes that the galaxy distribution traces the mass distribution. Although it seems reasonable that regions of high density of galaxies correspond to regions of high mass density, since most of the mass is dark, the proportionality might not be exact. Thus, we have to allow for a possible *bias* in the power spectrum. Other problems also arise. The distance to an object is not measured directly; what is measured is its redshift. The redshift is determined by the distance to the object, as well as its peculiar motion. In regions of large overdensity the peculiar motions may be large, resulting in what is known as redshift distortions. Another problem is nonlinear evolution, which distorts the power spectrum in regions of large overdensity. Thus, if one wants to compare the observed power spectrum with the linear power spectrum generated by early-universe physics, it is necessary to make corrections for bias, redshift-space distortions, and nonlinear evolution.

Deducing the power spectrum from galaxy surveys is a tricky business. Rather than go into the details, uncertainties, and all that, I will just present a representative power spectrum in Fig. (1). Note that $\delta(k)$ decreases with increasing length scale (decreasing wavenumber). The universe is lumpy on small scales, but becomes progressively smoother when examined on larger scales.

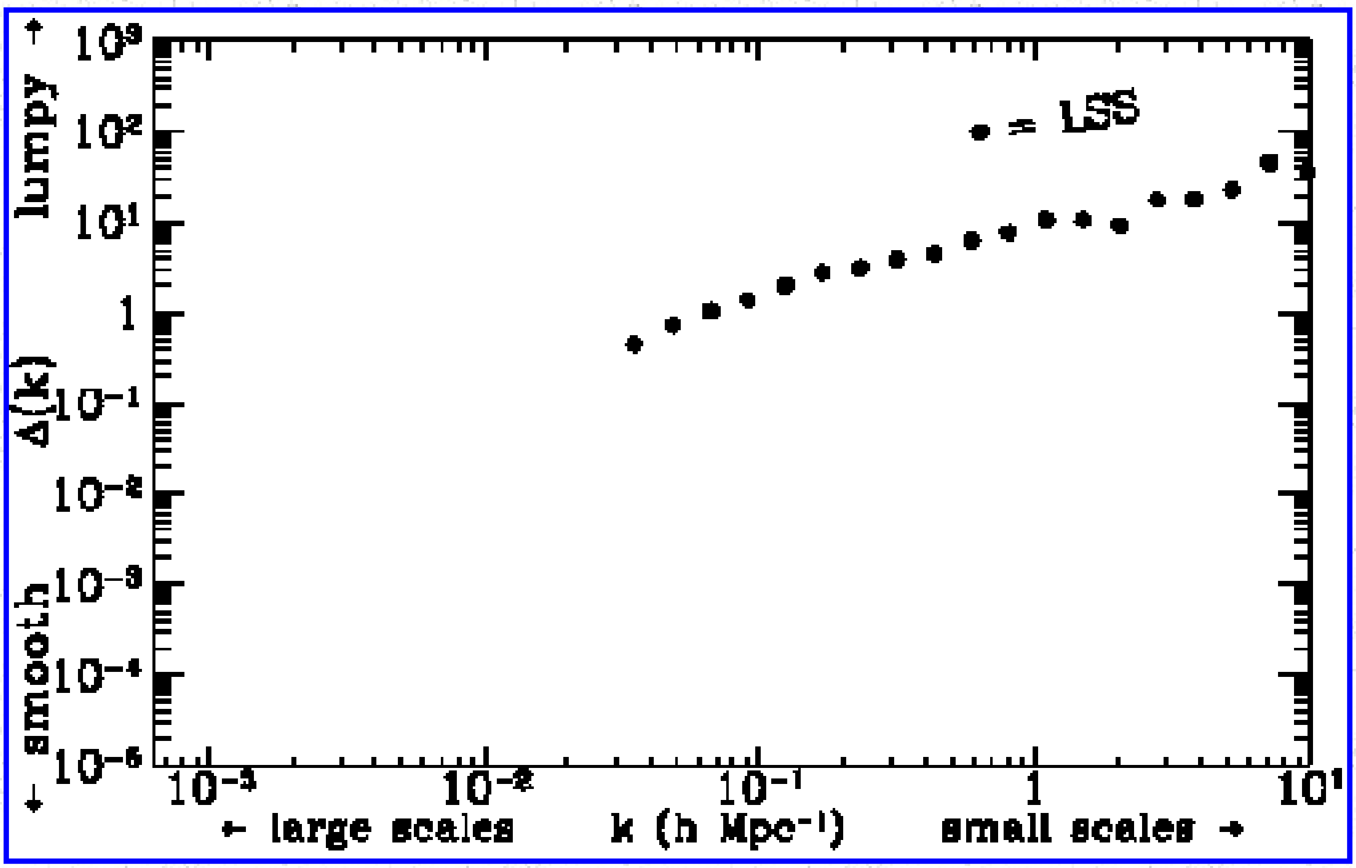

Figure 1. An example of a power spectrum deduced from a large-scale structure (LSS) survey. A megaparsec (Mpc) is 3.26 x 10^{24} cm, and h is the dimensionless Hubble constant, $H_0 = 100h$ km s^{-1} Mpc^{-1}. A wavenumber k is roughly related to a length scale of $2\pi / k$.

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2.2 The Power Spectrum Deduced From the Cosmic Background Radiation

The microwave background is isotropic to about one part in 10^3. If one removes the anisotropy caused by our motion with respect to the cosmic background radiation (CBR) rest frame, then it is isotropic to about 30 parts-per-million. But as first discovered by the Cosmic Background Explorer (COBE), there are intrinsic fluctuation in the temperature of the CBR.

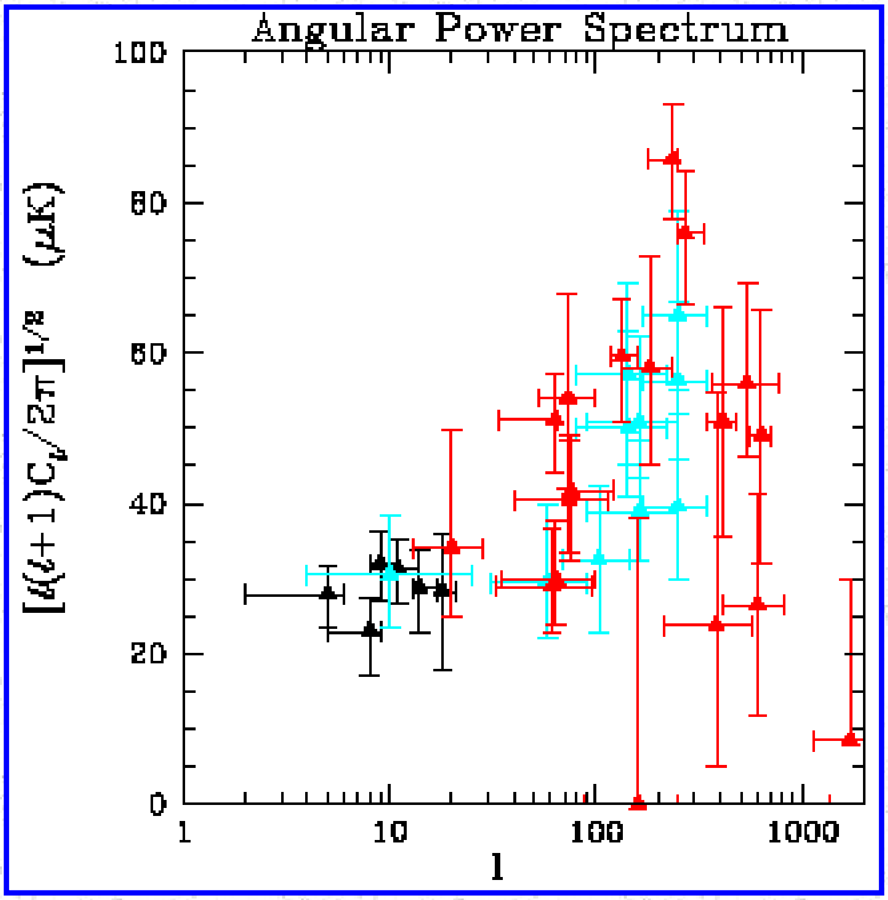

Figure 2. The angular power spectrum of CBR fluctuations (courtesy of Dick Bond and Llyod Knox).

Just as perturbations in the density field were expanded in terms of Fourier components, a similar expansion is useful for temperature fluctuations. Because the surface of observation about us can be described in terms of spherical angles θ and ϕ, the correct expansion basis is spherical harmonics, Y_{lm} (θ, ϕ). If the average temperature is $< T >$, then one can expand

$$\frac{\Delta T(\theta, \phi)}{\langle T \rangle} = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi) \ . \qquad (5)$$

Of course < a_{lm} > = 0, but with proper averaging,

$$\langle |a_{lm}|^2 \rangle \equiv C_l \neq 0 \; . \qquad (6)$$

C_l as a function of l is called the angular power spectrum. In the six years since the first measurement of CBR fluctuations by COBE, a number of experiments have detected fluctuations. The present situation is illustrated in Fig. 2.

Associated with a multipole number l is a characteristic angle θ, and a length scale we can define as the distance subtended by θ on the surface of last scattering. Since the distance to the last scattering surface of the microwave background is so large, the temperature fluctuations represent the largest structures ever seen in the universe.

Contributing to the temperature anisotropies are fluctuations in the gravitational potential on the surface of last scattering. Photons escaping from regions of high density will suffer a larger than average gravitational redshift, hence will appear to originate from a cold region. In similar fashion, photons coming to us from a low-density region will appear hot. In this manner, temperature fluctuations can probe the density field on the surface of last scattering and provide information about the power spectrum on scales much larger than can be probed by conventional large-scale structure observations.

The region of wavenumber and amplitude of the power spectrum probed by COBE is illustrated in Fig. 3. There are now measurements of CBR fluctuations on smaller angular scale, corresponding to larger k.

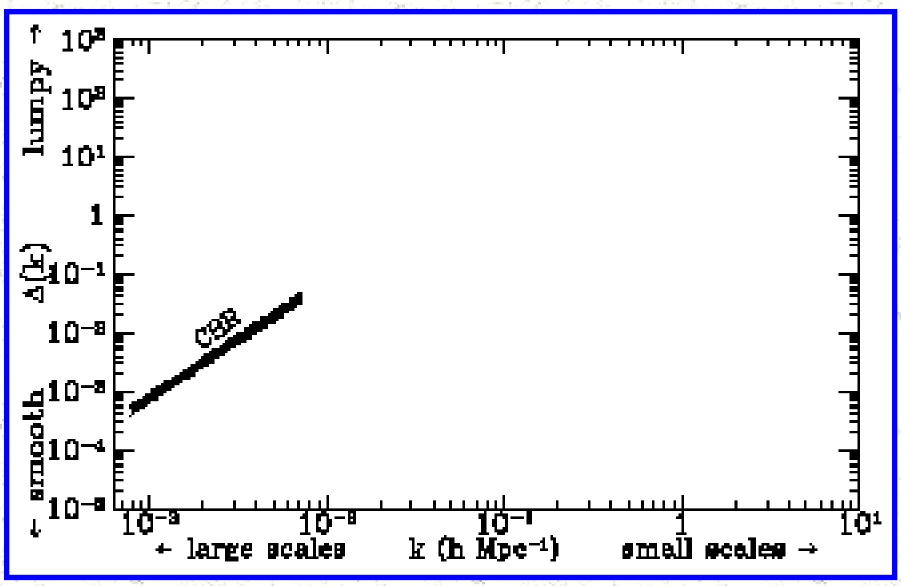

Figure 3. The power spectrum deduced by measurements of large angular scale CBR temperature fluctuations.

Finally, Fig. 4 combines information from both large-scale structure surveys and CBR temperature fluctuations. The trend is obvious: on small distance scales the power spectrum is ``large," which implies a lot of structure. Matter is clustered on small scales. But on ``large" scales the power spectrum decreases. As one examines the universe on larger scales, homogeneity and isotropy becomes a better and better approximation.

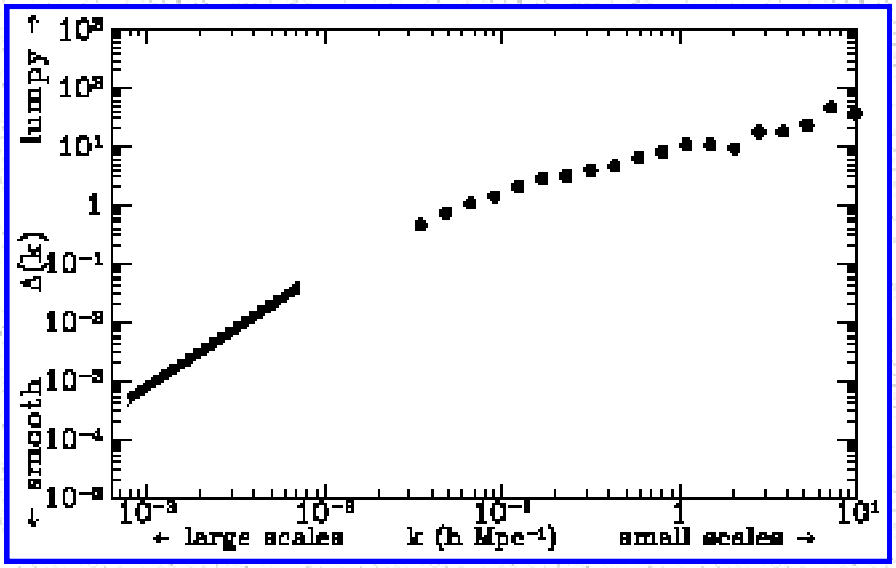

Figure 4. The ``grand unified" power spectrum, including determinations from large-scale structure surveys (the points), and deduced from CBR temperature fluctuations (the box).

The data shown is only illustrative of many data sets. Although combining different data sets is uncertain and risky (problems with normalization, etc.) the qualitative features are the same. Figure 4 is best regarded as an impressionist representation of the situation.

Another thing to keep in mind is that the power spectrum may not be the entire story. The power spectrum contains all statistical information about the perturbations only if the fluctuations are Gaussian. This should be cause for concern, because even if the initial perturbations are Gaussian, eventually they will become non-Gaussian once the perturbations become nonlinear. Also, the power spectrum is not a useful discriminant for prominent features such as walls, voids, filaments, etc. In spite of its drawbacks, the power spectrum is remarkably useful - if we can't get the power spectrum right, then we are not on the right track.

Now we turn to an early-universe theory that can account for the power spectrum: inflation

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3. INFLATION

One of the striking features of the CBR fluctuations is that they *appear* to be noncausal. The CBR fluctuations were largely imprinted at the time of last-scattering, about 300,000 years after the bang. However, there are fluctuations on length scales much larger than 3000,000 light years! How could a causal process imprint fluctuations on scales larger than the light-travel distance since the time of the bang? The answer is inflation, but to see how that works, let's define the problem more exactly.

First consider the evolution of the Hubble radius with the scale factor *a (t)*: [(1)]

$$R_H \equiv H^{-1} = \left(\frac{\dot{a}}{a}\right)^{-1} \propto \rho^{-1/2} \propto \begin{cases} a^2 & \text{(RD)} \\ a^{3/2} & \text{(MD).} \end{cases} \qquad (7)$$

In a $k = 0$ matter-dominated universe the age is related to H by $t = (2/3)H^{-1}$, so $R_H = (3/2)t$. In the early radiation-dominated universe $t = (1/2)H^{-1}$, so $R_H = 2t$.

On length scales smaller than R_H it is possible to move material around and make an imprint upon the universe. Scales larger than R_H are ``beyond the Hubble radius,'' and the expansion of the universe prevents the establishment of any perturbation on scales larger than R_H.

Next consider the evolution of some physical length scale λ. Clearly, any physical length scale changes in expansion in proportion to *a (t)*.

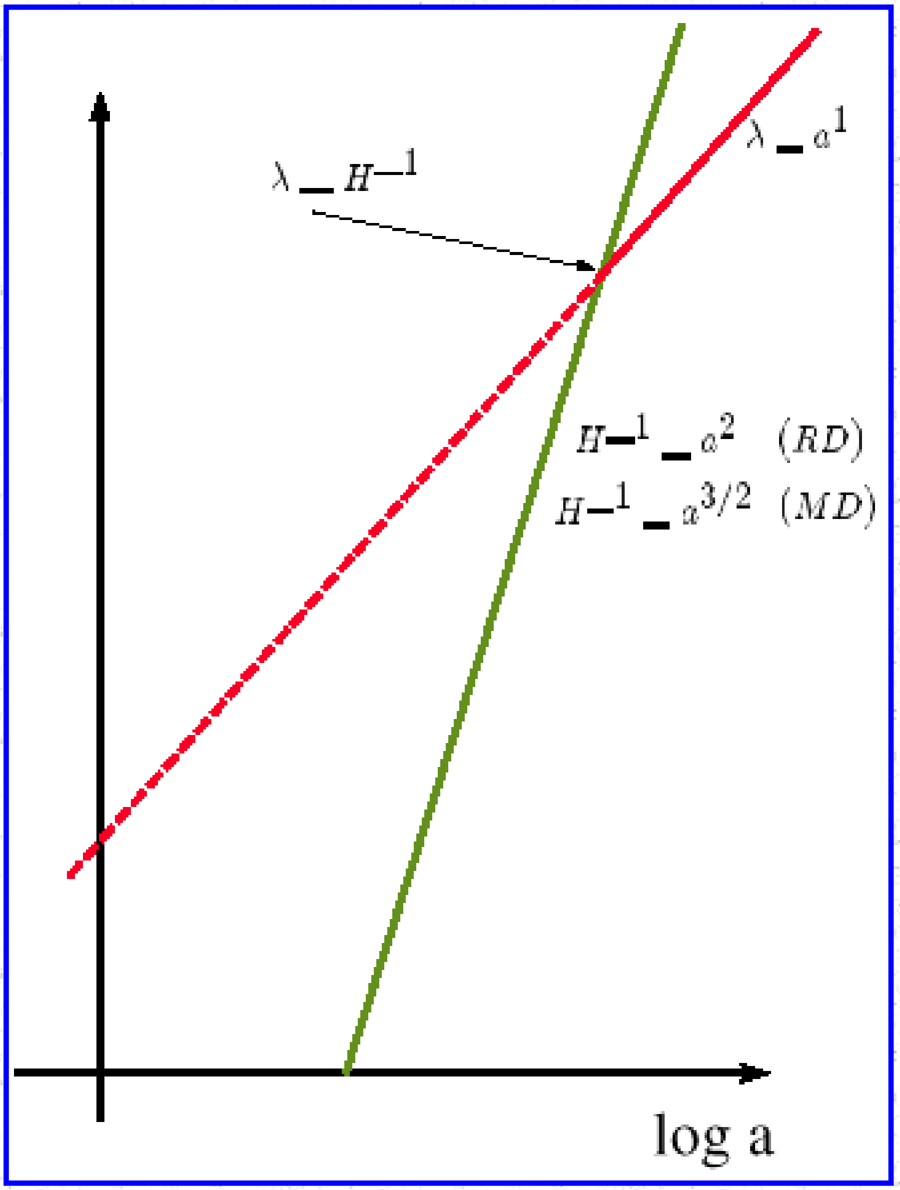
Now let us form the dimensionless ratio $L \equiv \lambda / R_H$. If L is smaller than unity, the length scale is smaller than the Hubble radius and it is possible to imagine some microphysical process establishing perturbations on that scale, while if L is larger than unity, no microphysical process can account for perturbations on that scale.

Since $R_H = a$ / dot$a$, and $\lambda \propto a$, the ratio L is proportional to dot$a$, and dot$L$ scales as dotdot$a$, which in turn is proportional to $-(\rho + 3p)$. There are two possible scenarios for dot$L$ depending upon the sign of $\rho + 3p$:

$$\dot{L} \begin{cases} < 0 \rightarrow & R_H \text{ grows faster than } \lambda, & \text{happens for } \rho + 3p > 0 \\ < 0 \rightarrow & R_H \text{ grows more slowly than } \lambda, & \text{happens for } \rho + 3p < 0. \end{cases} \qquad (8)$$

In the standard scenario, $\rho + 3p > 0$, R_H grows faster than λ. This is illustrated by the left-hand side of Fig. 5.

For illustration, let us take λ to be the present length $\lambda_8 = 8h^{-1}$ Mpc, the scale beyond which perturbations today are in the linear regime. The physical length scale, which today is λ_8, was smaller in the early universe by a factor of $a(t) / a_0$, where a_0 is the scale factor today. Today, the Hubble radius is $H_0^{-1} \sim 3000\ h^{-1}$ Mpc. Of course, today λ_8 is well within the current Hubble radius. But in the standard picture, R_H grows faster than λ, and there must therefore have been a time when the comoving length scale that corresponds to λ_8 was larger than R_H



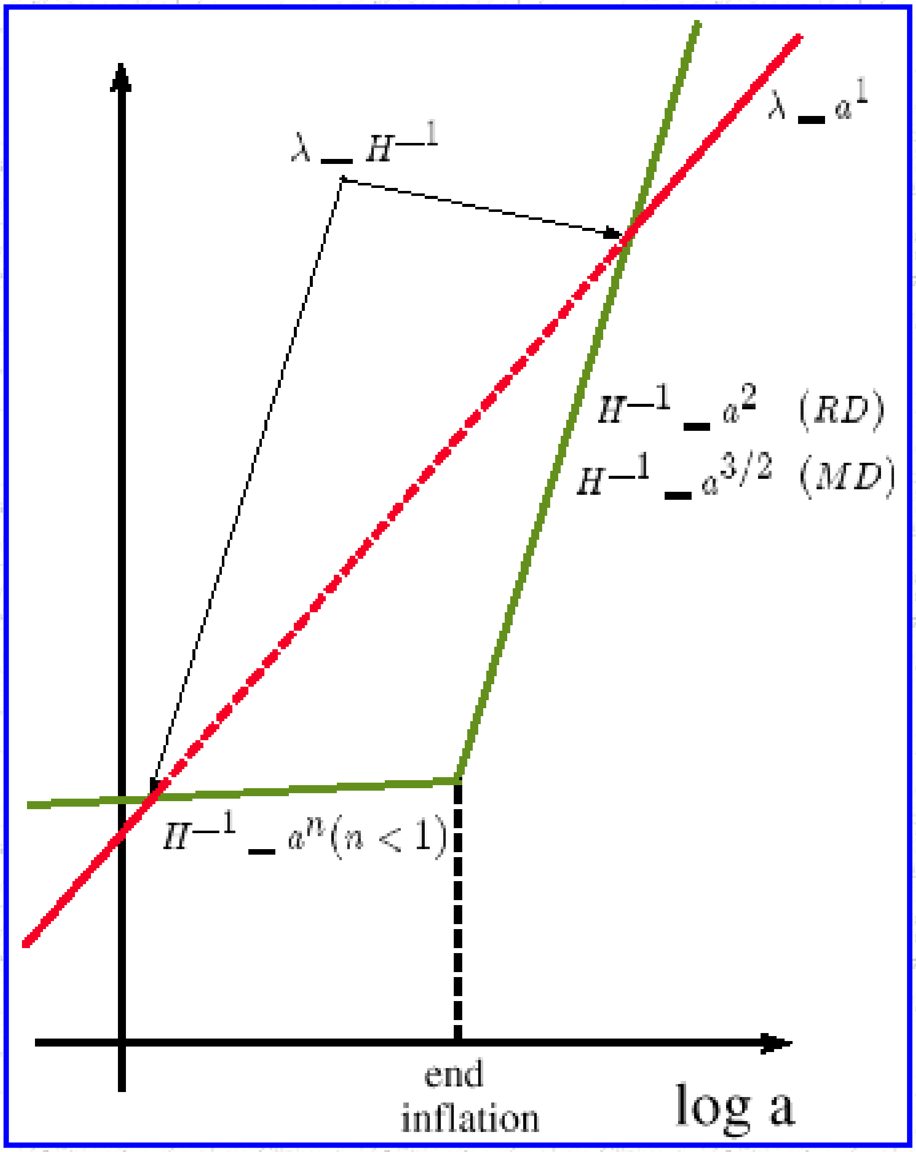


Figure 5. Physical sizes increase as *a (t)* in the expanding universe. The Hubble radius evolves as $R_H = H^{-1} = (8\pi G \rho(a) / 3)^{1/2}$. In a radiation-dominated or matter-dominated universe (left) any physical length scale λ starts larger than R_H, then crosses the Hubble radius ($\lambda = H^{-1}$) only once. However, if there was a period of early inflation (right) when R_H increased more slowly than a, it is possible for a physical length scale to start smaller than R_H, become larger than R_H, and after inflation ends become once again smaller than R_H. Periods during which the scale is larger than the Hubble radius are indicated by the dotted line.

Sometime during the early evolution of the universe the expansion was such that dotdot$a > 0$, which as we have just seen, requires an unusual equation of state with $\rho + 3p < 0$. This is referred to as ``accelerated expansion" or ``inflation."

Table 1. Different epochs in the history of the universe and the associated tempos of the expansion rate.

tempo	passage	age	ρ	p	$\rho + 3p$
prestissimo	string dominated	$< 10^{-43}$ s	?	?	?
presto	vacuum dominated (inflation)	$\sim 10^{-38}$ s	ρ_V	$-\rho_V$	−
allegro	matter dominated	$\sim 10^{-36}$ s	ρ_ϕ	0	+
andante	radiation dominated	$< 10^4$ yr	T^4	$T^{4/3}$	+
largo	matter dominated	$> 10^4$yr	ρ_{matter}	0	+

Including the inflationary phase, our best guess for the different epochs in the history of the universe is given in Table 1. There is basically nothing known about the stringy phase, if indeed there was one. The earliest phase we have information about is the inflationary phase. As we shall see, the information we have is from the quantum fluctuations during inflation, which were imprinted upon the metric, and can be observed as CBR fluctuations and the departures from homogeneity and isotropy in the matter distribution, e.g., the power spectrum. A lot of effort has gone into studying the end of inflation. It was likely that there was a brief period of matter domination before the universe became radiation dominated. Very little is known about this period after inflation. Noteworthy events that might have occurred during this phase include baryogenesis, phase transitions, and generation of dark matter. We do know that the universe was radiation dominated for almost all of the first 10,000 years. The best evidence of the radiation-dominated era is primordial nucleosynthesis, which is a relic of the radiation-dominated universe in the period 1 second to 3 minutes. The earliest picture of the matter-dominated era is the CBR.

Here, I am interested in events during the inflationary era. The first issue is how to imagine a universe dominated by vacuum energy making a transition to a matter-dominated or radiation-dominated universe. A simple way to picture this is by the action of a scalar field ϕ with potential $V(\phi)$. Let's imagine the scalar field is displaced from the minimum of its potential as illustrated in Fig. 6. If the energy density of the universe is dominated by the potential energy of the scalar field ϕ, known as the *inflaton*, then $\rho + 3p$ will be negative. The vacuum energy disappears when the scalar field evolves to its minimum.

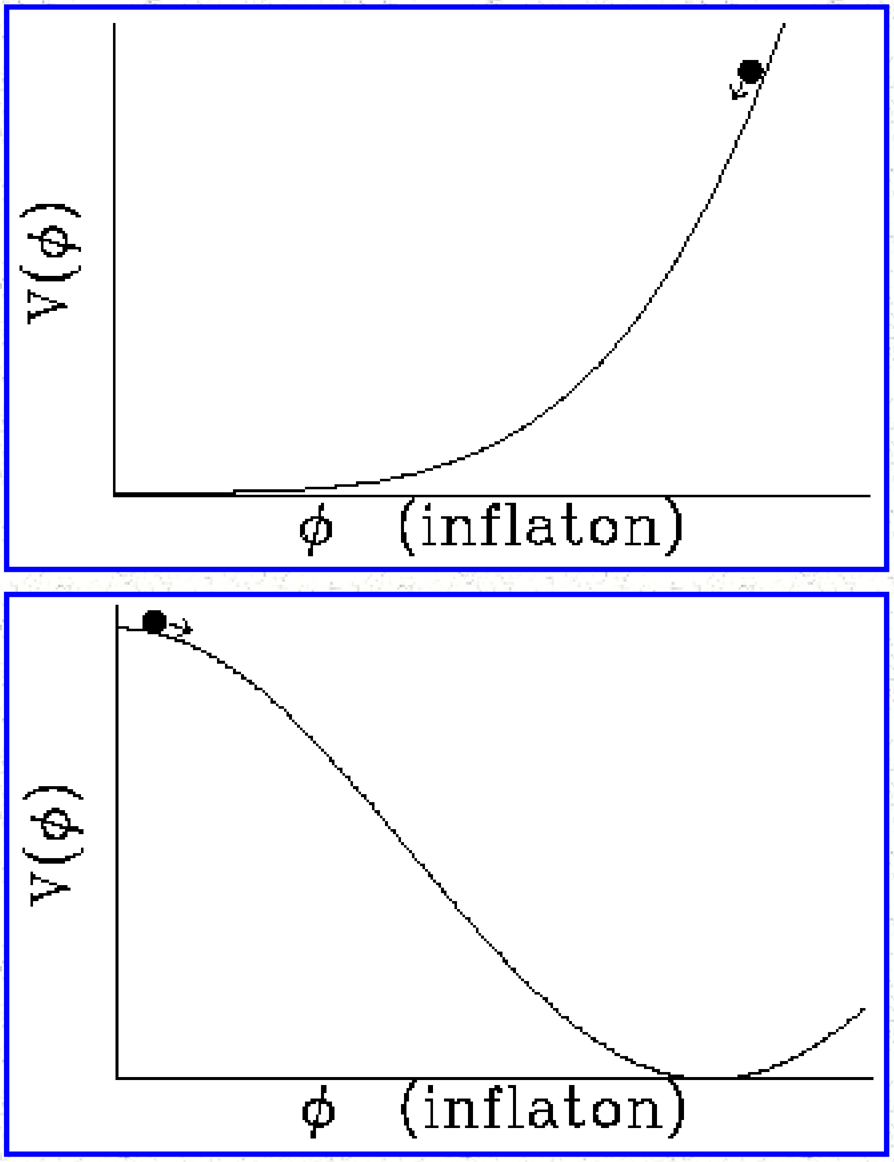


Figure 6. Schematic illustrations of the inflaton potential energy. The potential on the left is a ``large-field" model, where the inflaton field starts large and evolves toward its minimum. The right figure illustrates a ``small-field" model. A more accurate description of large-field and small-field potential is the sign of the second derivative of the potential: large-field models have $V'' > 0$ while small-field models have $V'' < 0$.

If the inflaton is completely decoupled, then it will oscillate about the minimum of the potential, with the cycle-average of the energy density decreasing as a^{-3}, i.e., a matter-dominated universe. But at the end of inflation the universe is cold and frozen in a low-entropy state: the only degree of freedom is the zero-momentum mode of the inflaton field. It is necessary to ``defrost" the universe and turn it into a ``hot" high-entropy universe with many degrees of freedom in the radiation. Exactly how this is accomplished is still unclear. It probably requires the inflaton field to be coupled to other degrees of freedom, and as it oscillates, its energy is converted to radiation either through incoherent decay, or through a coherent process involving very complicated dynamics of coupled oscillators with time-varying masses. In either case, it is necessary to extract the energy from the inflaton and convert it into radiation.

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(1) Here and throughout the paper ``RD'' is short for radiation dominated, and ``MD'' implies matter dominated. Back

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3.1 Quantum Fluctuations

During inflation there are quantum fluctuations in the inflaton field. Since the total energy density of the universe is dominated by the inflaton potential energy density, fluctuations in the inflaton field lead to fluctuations in the energy density. Because of the rapid expansion of the universe during inflation, these fluctuations in the energy density are frozen into super-Hubble-radius-size perturbations. Later, in the radiation or matter-dominated era they will come within the Hubble radius as if they were *noncausal* perturbations.

The spectrum and amplitude of perturbations depend upon the nature of the inflaton potential. Mukhanov [3] has developed a very nice formalism for the calculation of density perturbations. One starts with the action for gravity (the Einstein-Hilbert action) plus a minimally-coupled scalar inflaton field ϕ:

$$S = -\int d^4x \sqrt{-g}\left[\frac{m_{Pl}^2}{16\pi}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right] . \qquad (9)$$

Here R is the Ricci curvature scalar. Quantum fluctuations result in perturbations in the metric tensor

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{\mu\nu}^{FRW} + \delta g_{\mu\nu} \\ \phi &\rightarrow \phi_0 + \delta\phi \ , \end{aligned} \qquad (10)$$

where $g_{\mu\nu}^{FRW}$ is the Friedmann-Robertson-Walker metric, and $\phi_0(t)$ is the classical solution for the homogeneous, isotropic evolution of the inflaton. The action describing the dynamics of the small perturbations can be written as

$$\begin{aligned} \delta_2 S &= \frac{1}{2}\int d^4x \left[\partial_\mu u \partial^\mu u + z^{-1}\frac{d^2 z}{d\tau^2}\ u^2\right] ; \\ z &= a\dot{\phi}/H \ , \end{aligned} \qquad (11)$$

i.e., the action in conformal time τ ($d\tau^2 = a^2(t)\, dt^2$) for a scalar field in Minkowski space, with mass-squared $m_u^2 = -z^{-1}\, d^2 z / d\tau^2$. Here, the scalar field u is a combination of metric fluctuations $\delta g_{\mu\nu}$ and scalar field fluctuations $\delta\phi$. This scalar field is related to the amplitude of the density perturbation.

The simple matter of calculating the perturbation spectrum for a noninteracting scalar field in Minkowski space will give the amplitude and spectrum of the density perturbations. The problem is that the solution to the field equations depends upon the background field evolution through the dependence of the mass of the field upon z. Different choices for the inflaton potential $V(\phi)$ results in different background field evolutions, and hence, different spectra and amplitudes for the density perturbations.

Before proceeding, now is a useful time to remark that in addition to scalar density perturbations, there are also fluctuations in the transverse, traceless component of the spatial part of the metric. These fluctuations (known as tensor fluctuations) can be thought of as a background of gravitons.

Although the scalar and tensor spectra depend upon $V(\phi)$, for most potentials they can be characterized by Q_{RMS}^{PS} (the amplitude of the scalar and tensor spectra on large length scales added in quadrature), n (the scalar spectral index describing the best power-law fit of the primordial scalar spectrum), r (the ratio of the tensor-to-scalar contribution to C_2 in the angular power spectrum), and n_T (the tensor spectral index describing the best power-law fit of the primordial tensor spectrum). For single-field, slow-roll inflation models, there is a relationship between n_T and r, so in fact there are only three independent variables. Furthermore, the amplitude of the fluctuations often depends upon a free parameter in the potential, and the spectra are normalized by Q_{RMS}^{PS}. This leads to a characterization of a wide-range of inflaton potentials in terms of two numbers, n and r.

In addition to the primordial spectrum characterized by n and r, in order to compare to data it is necessary to specify cosmological parameters (H_0, the present expansion rate; ω_0, the ratio of the present mass-energy density to the critical density - a spatially flat universe has $\omega_0 = 1$; ω_B, the ratio of the present baryon density to the critical density; ω_{DM} the ratio of the present dark-matter density to the critical density; and λ, the value of the cosmological constant), as well as the nature of the dark matter.

The specification of the dark matter is by how ``hot" the dark matter was when the universe first became matter dominated. If the dark matter was really slow at that time, then it is referred to as cold dark matter. If the dark matter was reasonably hot when the universe became matter dominated, then it is called hot dark matter. Finally, the intermediate case is called warm dark matter. Neutrinos with a mass in the range 1 eV to a few dozen eV would be hot dark matter. Light gravitinos, as appear in gauge-mediated supersymmetry breaking schemes, is an example of warm dark matter. By far the most popular dark matter candidate is cold dark matter. Examples of cold dark matter are neutralinos and axions.

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4. THE FLAVOR OF THE MONTH

There are exactly 31 different combinations of *n*, *r*, cosmological parameters, and dark matter mixes. [(2)] For this reason, different cosmological models are like flavors of ice cream at Baskin Robbins. There is always a flavor of the month that everyone seems to like. Flavors come in and out of taste / fashion, with some adherents always choosing the same, while others like to sample a wide variety. A menu of the six most popular flavors is given in Table 2.

Table 2. Different flavors of cosmological models.

flavor	n	r	H_0	Ω_0	Ω_B	Ω_{COLD}	Ω_{HOT}	Ω_Λ
CDM	1	0	50 km s^{-1} Mpc^{-1}	1	0.05	0.95	0	0
HDM	1	0	50 km s^{-1} Mpc^{-1}	1	0.05	0	0.95	0
MDM	1	0	50 km s^{-1} Mpc^{-1}	1	0.05	0.70	0.20	0
TCDM	0.8	0	50 km s^{-1} Mpc^{-1}	1	0.05	0.95	0	0
OCDM	1	0	50 km s^{-1} Mpc^{-1}	0.5	0.05	0.45	0	0
ΛCDM	1	0	50 km s^{-1} Mpc^{-1}	1	0.05	0.45	0	0.50

Obviously, other combinations are possible. A comparison of the power spectrum in these models to our impressionist version of the observationally determined power spectrum is shown in Fig. 7. Obviously CDM has too much power on small scales. Hot dark matter is a disaster because it has *no* power on small scales. Tilted dark matter does better than CDM. Mixed dark matter does somewhat better, as does λ dark matter (not shown). Rather than χ-by-eye, I quote the results of one statistical analysis, including many data sets, in Table 3 [2].

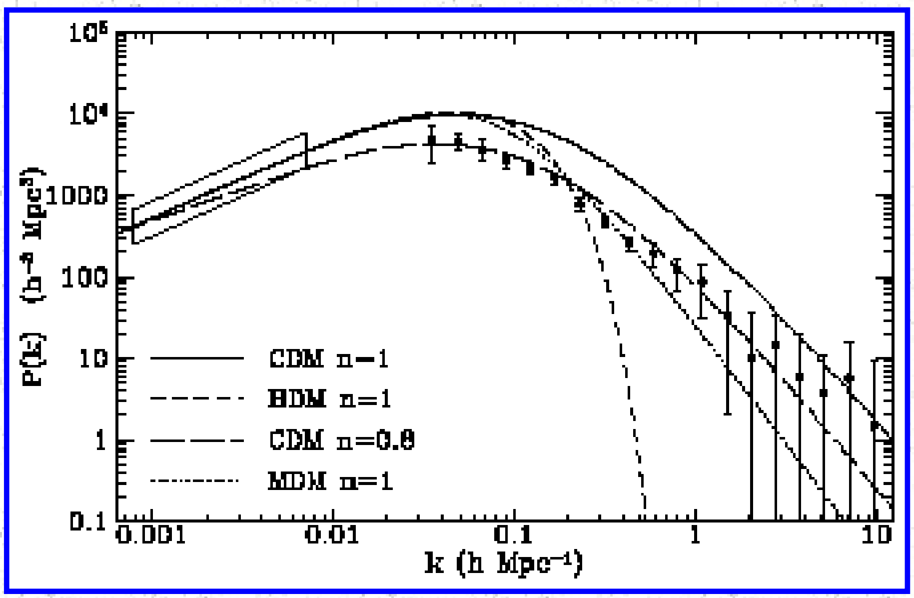

Figure 7. The empirically determined power spectrum of density perturbations and the (linear-theory) predictions of several models. The models shown are cold dark matter; hot dark matter; tilted cold dark matter; and mixed dark matter.

My reading of the comparison between data and experiment is that the results of Table 3 should be regarded as a *relative* measure of the agreement between models and present data. For instance, it is fair to say that MDM is a much better fit than CDM. But one should be very careful before rejecting a model based upon these numbers.

Table 3. One analysis of the comparison of data and models.

model	χ^2 / *d.o.f*
CDM	3.8
TCDM	2.1
ΛCDM	1.9
OCDM	1.8
MDM	1.2

Although one might get the best χ^2 with a model having 10% baryons, 30% cold dark matter, 30% hot dark matter with 15% each in two species of neutrinos, 20% cosmological constant, 10% warm dark mater, and seasoned with a little tilt, it doesn't mean that is the way the universe is constructed.

Clearly, what is needed are better observations: finer-scale observations of CBR fluctuations, as well as larger-scale determinations of the power spectrum from large-scale structure surveys. In the next few years such experiments will be done.

There is now an aggressive campaign to measure CBR anisotropies on fine angular scales. The culmination of this program will be the launch of two satellites - MAP by NASA and Planck by ESA.

Large-scale structure surveys are also progressing. The largest (three-dimensional) survey to date is the Las Campanas Redshift Survey, containing the three-dimensional location of over 30,000 galaxies. In the next two years another survey, called 2dF, will be completed. This survey will have about 100,000 galaxies in its catalog. Finally, the Sloan Digital Sky Survey will map π steradians of the north galactic cap and find the location of 1,000,000 galaxies, along with 150,000 quasars.

By the time these experiments/observations are complete we will be in the age of precision cosmology, and we should really be able to compare theory and observation.

The remaining missing piece of the puzzle may be the identity of the dark matter.

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[2] This statement clearly is not true. Back

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5. DARK MATTER

5.1 WIMPY Thermal Relics

In this school, the matter of neutrino masses has been reviewed in great detail. A neutrino of mass m_ν contributes to $\Omega_\nu h^2$ an amount

$$\Omega_\nu h^2 = \left(\frac{m_\nu}{92\mathrm{eV}}\right) . \qquad (12)$$

If the mass of the neutrino is significantly less than 0.1 eV, then its contribution to Ω_0 is dynamically unimportant.

More promising than neutrino hot dark matter is cold dark matter. The most promising candidate for cold dark matter is the lightest supersymmetric particle, presumably a neutralino. Neutralino dark matter has been well studied and reviewed [4].

The next most popular dark-matter candidate is the axion. Although the axion is very light, since its origin is from a condensate, it is very cold. Axion dark matter has also been well studied and well reviewed [5].

There are presently several experiments searching for cosmic neutralinos and cosmic axions. Both types of searches seem sensitive enough to discover the relic dark matter, although it will take quite some time (and probably another generation of experiments) to completely cover the parameter space.

Neutralinos are an example of a thermal relic. A thermal relic is assumed to be in local thermodynamic equilibrium (LTE) at early times. The *equilibrium* abundance of a particle, say relative to the entropy density, depends upon the ratio of the mass of the particle to the temperature. If we define the variables $Y \equiv n_X / s$ and $x = M_X / T$, where n_X is the number density of WIMP (weakly interacting massive particle) X with mass M_X and $s \sim T^3$ is the entropy density, $Y \propto \exp(-x)$ for $x >> 1$, while $Y \sim$ constant for $x << 1$.

A particle will track its equilibrium abundance so long as reactions which keep the particle in chemical equilibrium can proceed rapidly enough. Here, rapidly enough means on a timescale more rapid than the expansion rate of the universe H. When the reactions becomes slower than the expansion rate, then the particle can no longer track its equilibrium value and thereafter Y is constant. When this occurs, the particle is said to be ``frozen out." A schematic illustration of this is given in Fig. 8.

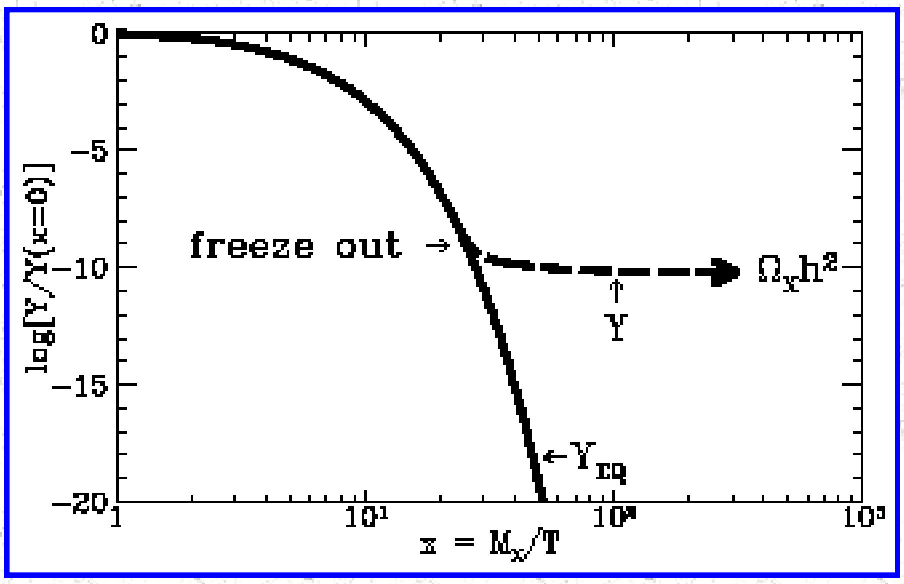

Figure 8. A thermal relic starts in LTE at $T >> M_X$. When the rates keeping the relic in chemical equilibrium become smaller than the expansion rate, the density of the relic relative to the entropy density becomes constant. This is known as *freeze out*.

The more strongly interacting the particle, the longer it stays in LTE, and the smaller its freeze-out abundance. Thus, the more weakly interacting the particle, the larger its present abundance. The freeze-out value of Y is related to the mass of the particle and its annihilation cross section (here characterized by σ_0) by

$$Y \propto \frac{1}{M_X m_{Pl} \sigma_0} \,. \tag{13}$$

Since the contribution to ω is proportional to $M_X\, n_X$, which in turn is proportional to $M_X Y$, the present contribution to ω from a thermal relic is (to first approximation) *independent* of the mass, and only depends on the mass indirectly through the dependence of the annihilation cross section on mass. The largest that the annihilation cross section can be is roughly M_X^{-2}. This implies that large-mass WIMPS would have such a small annihilation cross section that their present abundance would be too large. Thus, one expects a maximum mass for a thermal WIMP, which turns out to be a few hundred TeV.

The mass of WIMPS usually considered for dark matter run from a microvolt for axions to several dozen GeV for neutralinos. With the exception of massive magnetic monopoles, the possibility of dark matter particles of GUT-scale mass is not usually considered, because thermal relics of this mass would be expected to be over abundant by several orders of magnitude.

Recently, the idea that dark matter may be *supermassive* has received a lot of attention. Since wimpy little dark matter particles with mass less than a TeV are called WIMPS, dark matter particles of really hefty mass of 10^{12} to 10^{16} GeV seem to be more than WIMPS, so they are referred to as WIMPZILLAS.

5.2 WIMPZILLAS - Size Does Matter

The simple assumption that the dark matter (DM) is a thermal relic is surprisingly restrictive. The limit $\omega_X \lesssim 1$ implies that the mass of a DM relic must be less than about 500 TeV [6]. The standard lore is that the hunt for DM should concentrate on particles with mass of the order of the weak scale and with interactions with ordinary matter on the scale of the weak force. This has been the driving force behind the vast effort in DM detectors.

But recent developments in understanding how matter is created in the early universe suggests the possibility that DM might be naturally composed of *nonthermal* supermassive states. The supermassive dark matter (WIMPZILLA) X may have a mass many orders of magnitude larger than the weak scale, possibly as large as the Grand Unified Theory (GUT) scale. It is very intriguing that these considerations resurrect the possibility that the dark matter might be charged or even strongly interacting!

The second condition for WIMPZILLAS is that the particle must not have been in equilibrium when it froze out (i.e., it is not a thermal relic), otherwise ω_X would be larger than one. A sufficient condition for nonequilibrium is that the annihilation rate (per particle) must be smaller than the expansion rate: $n \sigma|v| < H$, where n is the number density, $\sigma|v|$ is the annihilation rate times the Møller flux factor, and H is the expansion rate. Conversely, if the WIMPZILLA was created at some temperature T_* *and* $\omega_X < 1$, then it is easy to show that it could not have attained equilibrium. To see this, assume Xs were created in a radiation-dominated universe at temperature T_*. Then ω_X is given by $\omega_X = \omega_\gamma (T_*/T_0) M_X n_X(T_*) / \rho_\gamma(T_*)$, where T_0 is the present temperature (ignoring dimensionless factors of order unity.) Using the fact that $\rho_\gamma(T_*) = H(T_*) M_{Pl} T_*^2$, we find $n_X(T_*) / H(T_*) = (\omega_X / \omega_\gamma) T_0 M_{Pl} T_* / M_X$. We may safely take the limit $\sigma|v| < M_X^{-2}$, so $n_X(T_*)\sigma|v| / H(T_*)$ must be less than $(\omega_X / \omega_\gamma) T_0 M_{Pl} T_* / M_X^3$. Thus, the requirement for nonequilibrium is

$$\left(\frac{200\,\mathrm{TeV}}{M_X}\right)^2 \left(\frac{T_*}{M_X}\right) < 1\,. \qquad (14)$$

This implies that if a nonrelativistic particle with $M_X \gtrsim 200$ TeV was created at $T_* < M_X$ with a density low enough to result in $\omega_X \lesssim 1$, then its abundance must have been so small that it never attained equilibrium. Therefore, if there is some way to create WIMPZILLAS in the correct abundance to give $\omega_X \sim 1$, nonequilibrium is guaranteed.

An attractive origin for WIMPZILLAS is during the defrosting phase after inflation. It is important to realize that it is not necessary to convert a significant fraction of the available energy into massive particles; in fact, it must be an infinitesimal amount. If a fraction ϵ of the available energy density is in the form of a massive, stable X particle, then $\omega_X = \varepsilon\, \omega_\gamma\, (T_{RH} / T_0)$, where T_{RH} is the ``reheat" temperature. For $\omega_X = 1$, this leads to the limit $\varepsilon \lesssim 10^{-17}\, (10^9\ \mathrm{GeV} / T_{RH})$.

In one extreme we might assume that the vacuum energy of inflation is immediately converted to radiation, resulting in a reheat temperature T_{RH}. In this case ω_X can be calculated by integrating the Boltzmann equation with initial condition $N_X = 0$ at $T = T_{RH}$. One expects the X density to be suppressed by $\exp(-2M_X / T_{RH})$; indeed, one finds $\omega_X \sim 1$ for $M_X / T_{RH} \sim 25 + 0.5 \ln(M_X^2 < \sigma |v| >)$, in agreement with previous estimates [7] that for $T_{RH} \sim 10^9$ GeV, the WIMPZILLA mass would be about 2.5×10^{10} GeV.

A second (and more plausible) scenario is that reheating is not instantaneous, but is the result of the decay of the inflaton field. In this approach the radiation is produced as the inflaton decays. The WIMPZILLA density is found by solving the coupled system of equations for the inflaton field energy, the radiation density, and the WIMPZILLA mass density. The calculation has been recently reported in Ref. [8], with result $\omega_X \sim M_X^2 < \sigma |v| > (2000\, T_{RH} / M_X)^7$. For a reheat temperature as low as 10^9 GeV, a particle of mass 10^{13} GeV can be produced in sufficient abundance to give $\omega_X \sim 1$.

The large difference in WIMPZILLA masses in the two reheating scenarios arises because the peak temperature is much larger in the second scenario, even with identical T_{RH}. Because the temperature decreases as $a^{-3/8}$ (a is the scale factor) during most of the reheating period in the second scenario, it must have once been much greater than T_{RH}. The evolution of the temperature is given in Fig. 9. If we assume the radiation spectrum did not depart grossly from thermal, the effective temperature having once been larger than T_{RH} implies that the density of particles with enough energy to create WIMPZILLAS was larger. Denoting as T^2 the maximum effective temperature for the second scenario, we find $T_2 / T_{RH} \sim (M_\phi / \gamma_\phi)^{1/4} >> 1$, where γ_ϕ is the effective decay rate of the inflaton. See [8] for details.

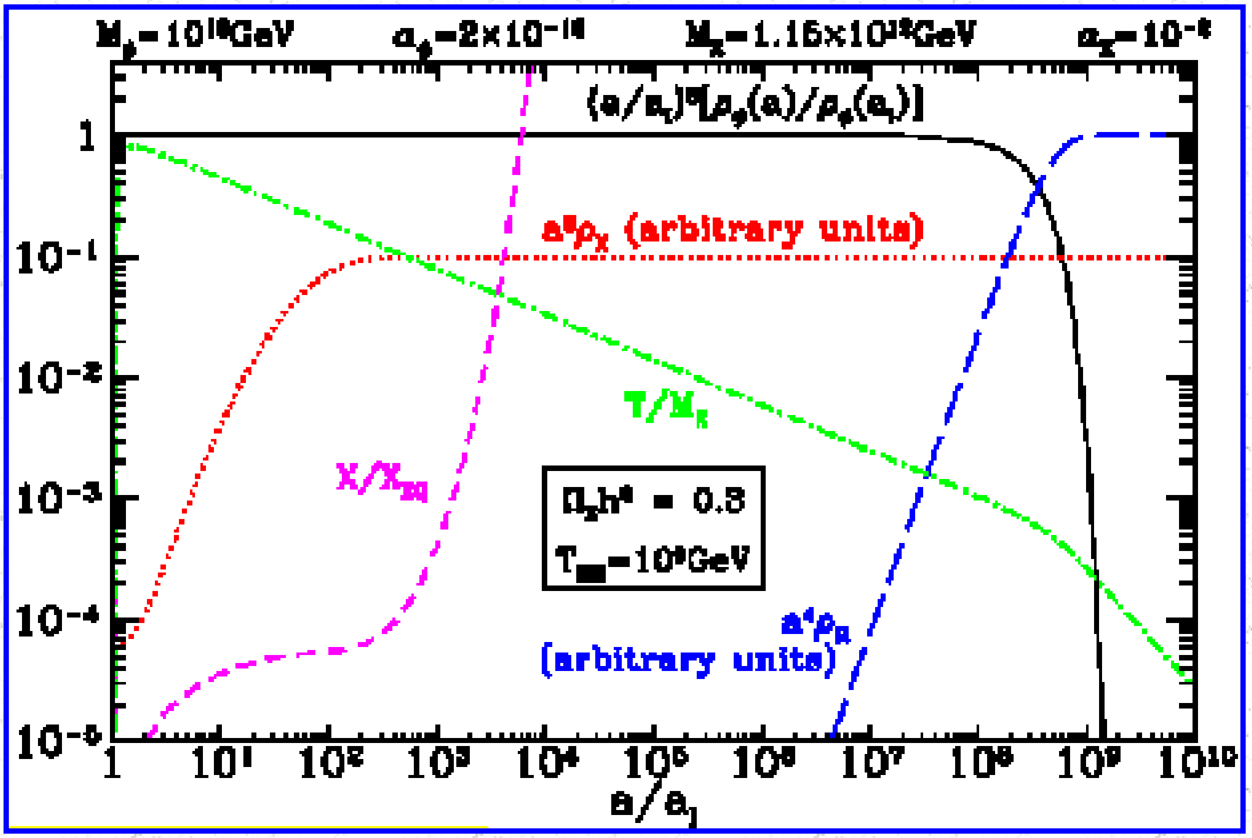

Figure 9. The evolution of energy densities and T/M_X as a function of the scale factor. Also shown is X/X_{EQ}.

Another way to produce WIMPZILLAS after inflation is in a preliminary stage of reheating called ``preheating'' [9], where nonlinear quantum effects may lead to an extremely effective dissipational dynamics and explosive particle production. Particles can be created in a broad parametric resonance with a fraction of the energy stored in the form of coherent inflaton oscillations at the end of inflation released after only a dozen oscillation periods. A crucial observation for our discussion is that particles with mass up to 10^{15} GeV may be created during preheating [10, 11, 12], and that their distribution is nonthermal. If these particles are stable, they may be good candidates for WIMPZILLAS.

To study how the creation of WIMPZILLAS takes place in preheating, let us take the simplest chaotic inflation potential: $V(\phi) = M\phi^2 \phi^2 / 2$ with $M\phi \sim 10^{13}$ GeV. We assume that the interaction term between the WIMPZILLA and the inflaton field is of the type $g^2\phi^2 |X|^2$. Quantum fluctuations of the X field with momentum $\vec{k}$ during preheating *approximately* obey the Mathieu equation, $X''_k + [A(k) - 2q \cos 2z] X_k = 0$, where $q = g^2 \phi^2 / 4 M\phi^2$, $A(k) = (k^2 + M_X^2) / M\phi^2 + 2q$ (primes denotes differentiation with respect to $z = M\phi\, t$). Particle production occurs above the line $A = 2q$ in an instability strip of width scaling as $q^{1/2}$ for large q. The condition for broad resonance, $A - 2q \lesssim q^{1/2}$ [10], becomes $(k^2 + M^2_X) / M\phi^2 \lesssim g\, bar\phi / M\phi$, which yields $E_X^2 = k^2 + M_X^2 \lesssim g\, bar\phi\, M\phi$ for the typical energy of particles produced in preheating. Here $bar\phi$ is the amplitude of the oscillating inflaton field [9]. The resulting estimate for the typical energy of particles at the end of the broad resonance regime for $M\phi \sim 10^{-6} M_{Pl}$ is

$E_X \sim 10^{-1} g^{1/2}$ sqrt $(M_\phi M_{\rm Pl}) \sim g^{1/2} 10^{15}$ GeV. Supermassive X bosons can be produced by the broad parametric resonance for $E_X > M_X$, which leads to the estimate that X production will be possible if $M_X < g^{1/2} 10^{15}$ GeV. For $g^2 \sim 1$ one would have copious production of X particles as heavy as 10^{15} GeV, i.e., 100 times greater than the inflaton mass, which may be many orders of magnitude greater than the reheat temperature. Scatterings of X fluctuations off the zero mode of the inflaton field considerably limits the maximum magnitude of X fluctuations to be $< X^2 >_{\rm max} \approx M_\phi^2 / g^2$ [13]. For example, $< X^2 >_{\rm max} \lesssim 10^{-10} M_{\rm Pl}^2$ in the case $M_X = 10 M_\phi$. This restricts the corresponding number density of created X-particles.

For a reheating temperature of the order of 100 GeV, the present abundance of WIMPZILLAS with mass $M_X \sim 10^{14}$ GeV is given by $\omega_X \sim 1$ if $\varepsilon \sim 10^{-10}$. This small fraction corresponds to $< X^2 > \sim 10^{-12} M_{\rm Pl}^2$ at the end of the preheating stage, a value naturally achieved for WIMPZILLA mass in the GUT range [13]. The creation of WIMPZILLAS through preheating and, therefore, the prediction of the present value of ω_X, is very model dependent. The inflaton might preferably decay through parametric resonance into very light boson fields so that the end of the preheating stage and of the corresponding value of $< X^2 >$ depends upon the coupling of the inflaton field not only to the WIMPZILLA, but also to other degrees of freedom. It is encouraging, however, that it is possible to produce supermassive particles during preheating that are as massive as $10^{12} T_{RH}$. Details of WIMPZILLA production in preheating can be found in [14].

Another possibility which has been recently investigated is the production of very massive particles by gravitational mechanisms [15, 16]. In particular, the desired abundance of WIMPZILLAS may be generated during the transition from the inflationary phase to a matter/radiation dominated phase as the result of the expansion of the background spacetime acting on vacuum quantum fluctuations of the dark matter field [15]. A crucial side-effect of the inflationary scenarios is the generation of density perturbations. A related effect, which does not seem to have attracted much attention, is the possibility of producing matter fields due to the rapid change in the evolution of the scale factor around the end of inflation. Contrary to the first effect, the second one contributes to the homogeneous background energy density that drives the cosmic expansion, and is essentially the familiar ``particle production'' effect of relativistic field theory in external fields.

Very massive particles may be created in a nonthermal state with sufficient abundance to achieve critical density today by the classical gravitational effect on the vacuum state at the end of inflation. Mechanically, the particle creation scenario is similar to the inflationary generation of gravitational perturbations that seed the formation of large-scale structures. However, the quantum generation of energy density fluctuations from inflation is associated with the inflaton field, which dominated the mass density of the universe, and not a generic sub-dominant scalar field.

If $0.04 \lesssim M_X / H_e \lesssim 2$ [15], where H_e is the Hubble constant at the end of inflation, DM produced gravitationally can have a density today of the order of the critical density. This result is quite robust with respect to the fine details of the transition between the inflationary phase and the matter-dominated phase. The only requirement is that

$$\left(\frac{H_e}{10^{-6}M_{Pl}}\right)^2\left(\frac{T_{RH}}{10^9\mathrm{GeV}}\right) \gtrsim 10^{-2}\ . \qquad (15)$$

The observation of anisotropy in the cosmic background radiation does not fix H_e uniquely, but using $T_{RH} \lesssim$ sqrt $(M_{Pl}\, H_e)$, we find that the mechanism is effective only when $H_e \gtrsim 10^9$ GeV (or, $M_X \gtrsim 10^8$ GeV).

The distinguishing feature of this mechanism [15] is the capability of generating particles with mass of the order of the inflaton mass even when the WIMPZILLA interacts only extremely weakly (or not at all!) with other particles, including the inflaton. This feature makes the gravitational production mechanism quite model independent and, therefore, more appealing to us than the one occurring at preheating.

WIMPZILLAS can also be produced in theories where inflation is completed by a first-order phase transition [17]. In these scenarios, the universe decays from its false vacuum state by bubble nucleation [18]. When bubbles form, the energy of the false vacuum is entirely transformed into potential energy in the bubble walls, but as the bubbles expand, more and more of their energy becomes kinetic and the walls become highly relativistic. Eventually the bubble walls collide.

During collisions, the walls oscillate through each other [19] and the kinetic energy is dispersed into low-energy scalar waves [19, 20]. If these soft scalar quanta carry quantum numbers associated with some spontaneously broken symmetry, they may even lead to the phenomenon of nonthermal symmetry restoration [21]. We are, however, more interested in the fate of the potential energy of the walls, $M_P = 4\pi\eta R^2$, where η is the energy per unit area of the bubble with radius R. The bubble walls can be imagined as a coherent state of inflaton particles, so that the typical energy E of the products of their decays is simply the inverse thickness of the wall, $E \sim \delta^{-1}$. If the bubble walls are highly relativistic when they collide, there is the possibility of quantum production of nonthermal particles with mass well above the mass of the inflaton field, up to energy $\delta^{-1} = \gamma M_\phi$, γ being the relativistic Lorentz factor.

Suppose now that the WIMPZILLA is some fermionic degree of freedom X and that it couples to the inflaton field by the Yukawa coupling $g\,\phi\,\bar{X} X$. One can treat ϕ (the bubbles or walls) as a classical, external field and the WIMPZILLA as a quantum field in the presence of this source. This amounts to ignoring the backreaction of particle production on the evolution of the walls, but this is certainly a good approximation in our case. The number of WIMPZILLA particles created in the collisions from the wall's potential energy is $N_X \sim f_X\, M_P / M_X$, where f_X parametrizes the fraction of the primary decay products that are WIMPZILLAS. The fraction f_X will depend in general on the masses and the couplings of a particular theory in question. For the Yukawa coupling g, it is $f_X \simeq g^2 \ln(\gamma M_\phi / 2 M_X)$ [20, 22]. Supermassive particles in bubble collisions are produced out of equilibrium and they never attain chemical equilibrium. Assuming $T_{RH} \simeq 100$ GeV, the present abundance of WIMPZILLAS is $\omega_X \sim 1$ if $g \sim 10^{-5} \alpha^{1/2}$. Here $\alpha^{-1} << 1$ denotes the fraction of the bubble energy at nucleation which has remained in the form of potential energy at the time of collision. This simple analysis indicates that the correct magnitude for the abundance of X particles may be naturally obtained in the process of reheating in theories where inflation is terminated by bubble nucleation.

In conclusion, a large fraction of the DM in the universe may be made of WIMPZILLAS of mass greatly exceed the electroweak scale - perhaps as large as the GUT scale. This is made possible by the fact that the WIMPZILLAS were created in a nonthermal state and never reached chemical equilibrium with the primordial plasma.

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