

# Cosmic Inflation\*

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## Abstract

I review the current status of the theory of Cosmic Inflation. My discussion covers the motivation and implementation of the idea, as well as an analysis of recent successes and open questions. There is a special discussion of the physics of “cosmic coherence” in the primordial perturbations. The issue of coherence is at the heart of much of the success inflation theory has achieved at the hands of the new microwave background data. While much of this review should be useful to anyone seeking to update their knowledge of inflation theory, I have also made a point of including basic material directed at students just starting research.

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## 1 Introduction

It has been almost 20 years since Guth's original paper fueled considerable excitement in the cosmology community [1]. It was through this paper that many cosmologists saw the first glimmer of hope that certain deep mysteries of the Universe could actually be resolved through the idea of Cosmic Inflation. Today inflation theory has come a long way. While at first the driving forces were deep theoretical questions, now the most exciting developments have come at the hands of new cosmological observations. In the intervening period we have learned a lot about the predictions inflation makes for the state of the

Universe today, and new technology is allowing these predictions to be tested to an ever growing degree of precision. The continual confrontation with new observational data has produced a string of striking successes for inflation which have increased our confidence that we are on the right track. Even so, there are key unanswered questions about the foundations of inflation which become ever more compelling in the face of growing observational successes.

## 2 Motivating Inflation: The Cosmological Features/Problems

It turns out that most of the cosmological “problems” that are usually introduced as a motivation for inflation are actually only “problems” if you take a very special perspective. I will take extra care to be clear about this by first presenting these issues simply as “features” of the standard Big Bang, and then discussing circumstances under which these features can be regarded as “problems”.

### 2.1 The Standard Big Bang

To start with, I will introduce the basic tools and ideas of the standard “Big Bang” (SBB) cosmology. Students needing further background should consult a text such as Kolb and Turner [2]. An extensive discussion of inflation can be found in [3]. The SBB treats a nearly perfectly homogeneous and isotropic universe, which gives a good fit to present observations. The single dynamical parameter describing the broad features of the SBB is the “scale factor”  $a$ , which obeys the “Friedmann equation”

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3}\rho - \frac{k}{a^2} \quad (1)$$

in units where  $M_P = \hbar = c = 1$ ,  $\rho$  is the energy density and  $k$  is the curvature. The Friedmann equation can be solved for  $a(t)$  once  $\rho(a)$  is determined. This can be done using local energy conservation, which for the SBB cosmology reads

$$\frac{d}{da}(\rho a^3) = -3p \frac{d}{da} a^3. \quad (2)$$

Here (in the comoving frame) the stress energy tensor of the matter is given by

$$T_{\nu}^{\mu} = \text{Diag}(\rho, p, p, p). \quad (3)$$

In the SBB, the Universe is first dominated by relativistic matter (“radiation dominated”) with  $w = 1/3$  which gives  $\rho \propto a^{-4}$  and  $a \propto t^{1/2}$ . Later the Universe is dominated by non-relativistic matter (“matter dominated”) with  $w = 0$  which gives  $\rho \propto a^{-3}$  and  $a \propto t^{2/3}$ .

The scale factor measures the overall expansion of the Universe (it doubles in size as the separation between distant objects doubles). Current data suggests an additional “Cosmological Constant” term  $\equiv \Lambda/3$  might be present on the right hand side of the Friedmann equation with a size similar to the other terms. However with the  $\rho$  and  $k$  terms evolving as negative powers of  $a$  these terms completely dominate over  $\Lambda$  at the earlier epochs we are discussing here. We set  $\Lambda = 0$  for the rest of this article.

## 2.2 The Flatness Feature

The “critical density”,  $\rho_c$ , is defined by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho_c. \quad (4)$$

A universe with  $k = 0$  has  $\rho = \rho_c$  and is said to be “flat”. It is useful to define the dimensionless density parameter

$$\Omega \equiv \frac{\rho}{\rho_c}. \quad (5)$$

If  $\Omega$  is close to unity the  $\rho$  term dominates in the Friedmann equation and the Universe is nearly flat. If  $\Omega$  deviates significantly from unity the  $k$  term (the “curvature”) is dominant.

The Flatness feature stems from the fact that  $\Omega = 1$  is an unstable point in the evolution of the Universe. Because  $\rho \propto a^{-3}$  or  $a^{-4}$  throughout the history of the Universe, the  $\rho$  term in the Friedmann equation falls away much more quickly than the  $k/a^2$  term as the Universe expands, and the  $k/a^2$  comes to dominate. This behavior is illustrated in Fig. 1.

Despite the strong tendency for the equations to drive the Universe away from critical density, the value of  $\Omega$  today is remarkably close to unity even after 15 Billion years of evolution. Today the value of  $\Omega$  is within an order of magnitude of unity, and that means that at early times  $\rho$  must have taken values that were set extremely closely to  $\rho_c$ . For example at the epoch of Grand Unified Theories (GUTs) ( $T \approx 10^{16} GeV$ ),  $\rho$  has to equal  $\rho_c$  to around 55 decimal places. This is simply an important property  $\rho$  must have for the SBB to fit the current observations, and at this point we merely take it as a feature (the “Flatness Feature”) of the SBB.

## 2.3 The Homogeneity Feature

The Universe today is strikingly homogeneous on large scales. Observations of the Cosmic Microwave Background (CMB) anisotropies show that the Universe was even more inhomogeneous in the distant past. Because the processes of gravitational collapse causes inhomogeneities to *grow* with time, an even greater degree of homogeneity must have been present at earlier epochs.

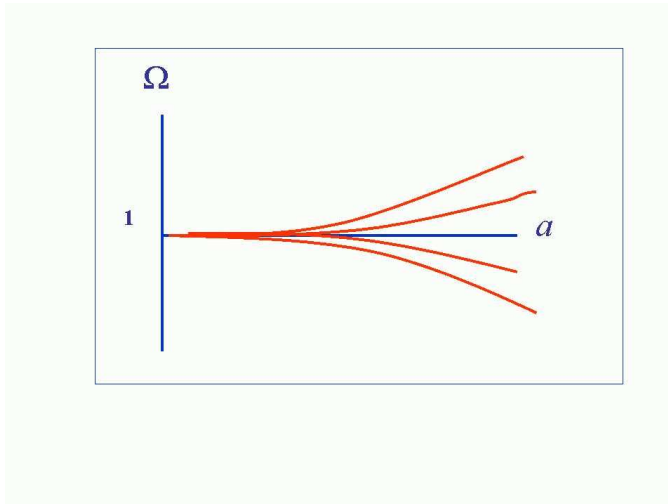


Figure 1: In the SBB  $\Omega(a)$  tends to evolve away from unity as the Universe expands

As with the flatness feature, the equations are driving the Universe into a state (very inhomogeneous) that is very different from the way it is observed today. This difference is present despite the fact that the equations have had a very long time to act. To arrange this situation, very special (highly homogeneous) initial conditions must be chosen, so that excessive inhomogeneities are not produced.

One might note that at early times the Universe was a hot relativistic plasma in local equilibrium and ask: Why can't equilibration processes smooth out any fluctuations and produce the necessary homogeneity? This idea does not work because equilibration and equilibrium are only possible on scales smaller than the "Jeans Length" ( $R_J$ ). On larger scales gravitational collapse wins over pressure and drives the system into an inhomogeneous non-equilibrium state.

At early epochs the region of the Universe we observe today occupied many Jeans volumes. For example, at the epoch of Grand Unification the observed Universe contained around  $10^{80}$  Jeans volumes (each of which would correspond to a scale of only several *cm* today). So the tendency toward gravitational collapse has had a chance to have a great impact on the Universe over the course of its history. Only through extremely precisely determined initial conditions can the amplitudes of the resulting inhomogeneities be brought down to acceptable levels. At the GUT epoch, for example, initial density contrasts

$$\delta(x) \equiv (\rho(x) - \bar{\rho})/\bar{\rho} \tag{6}$$

that are non-zero only in the 20th, 40th or even later decimal places, depending

on the scale, are required to arrange the correct degree of homogeneity. So here we have another special feature, the “Homogeneity Feature” which is required to make the SBB consistent with the Universe that we observe.

## 2.4 The Horizon Feature

One can take any physical system and choose an “initial time”  $t_i$ . The state of the system at any later time is affected both by the state at  $t_i$  (the initial conditions) and by the subsequent evolution. At any finite time after  $t_i$  there are causal limits on how large a scale can be affected by the subsequent evolution (limited ultimately by the speed of light, but really by the actual propagation speeds in that particularly system, which can be much slower). So there are always sufficiently large scales which have not been affected by the subsequent evolution, and on which the state of the system is simply a reflection of the initial conditions.

This same concept can equally well be applied to the SBB. However, there is one very interesting difference: The SBB starts with an initial singularity where  $a = 0$  and  $\rho = \infty$  at a finite time in the past. This is a serious enough singularity that the equations cannot propagate the Universe through it. In the SBB the Universe must simply start at this initial singularity with chosen initial conditions. So while in a laboratory situation the definition of  $t_i$  might be arbitrary in many cases, there is an absolutely defined  $t_i$  in the SBB.

As with any system, the causal “horizon” of the Universe grows with time. Today, the region with which we are just coming into causal contact (by observing distant points in the Universe) is one “causal radius” in size, which means objects we see in opposite directions are two causal radii apart and have not yet come into causal contact. One can calculate the number of causal regions that filled the currently observed Universe at other times. In the early Universe the size of a causally connected region was roughly the Jeans length, and so at the Grand Unification epoch there were around  $10^{80}$  causally disconnected regions in the volume that would evolve into the part of the Universe we currently can observe. This is the “Horizon Feature” of the SBB, and is depicted in Fig. 2

## 2.5 When the Features become Problems

I have discussed three features of the SBB that are usually presented as “problems”. What can make a feature a problem? The Flatness and Homogeneity features describe the need to give the SBB very special initial conditions in order to match current data. Who cares? If you look at the typical laboratory comparison between theory and experiment, success is usually measured by whether or not theoretical equations of motion correctly describe the evolution of the system. The choice of initial conditions is usually made only in order to facilitate the comparison. By these standards, the SBB does not have any

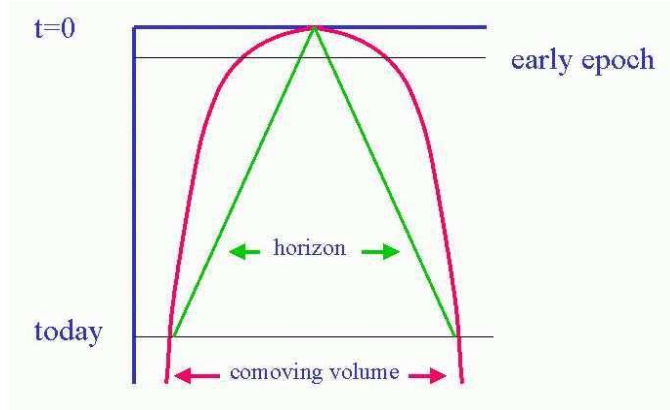


Figure 2: The region of the Universe we see today was composed of many causally disconnected regions in the past.

problems. The equations, with suitably chosen initial conditions, do a perfectly good job of describing the evolution of the Universe.

However, there is something extremely strange about the initial conditions required by the SBB that makes us unable to simply accept the Flatness and Homogeneity features: The required initial conditions place the Universe very far away from where its equations of motion wish to take it. So much so that even today, 15 Billion years later, the Universe still has features (flatness and homogeneity) that the equations are trying to destroy.

I think a simple analogy is useful: Imagine that you come into my office and see a pencil balanced on its point. You notice it is still there the next day, the next week, and even a year later. You finally ask me “what is going on with the pencil?” to which I reply: “yes, that’s pretty interesting, but I don’t have to explain it because it was that way when I moved into my office”. You would find such a response completely unreasonable. On the other hand, if the pencil was simply lying on my desk, you probably would not care at all how it got there.

The fact is that, like the fictitious balanced pencil, the initial conditions of the real Universe are so amazing that we cannot tear ourselves away from trying to understand what created them. It is at this point that the Flatness and Homogeneity features become “problems”.

It is also at this point that the Horizon feature becomes a problem. The hope of a cosmologist who is trying to explain the special initial conditions is that some new physical processes can be identified which can “set up” these initial conditions. A natural place to hope for this new physics to occur is the Grand Unification (GUT) scale, where particle physics already indicates that something new will happen. However, once one starts thinking that way, one

faces the fact that the Universe is composed of  $10^{80}$  causally disconnected regions at the GUT epoch, and it seems impossible for physical processes to extend across these many disconnected regions and set up the special homogeneous initial conditions.

## 2.6 The Monopole Problem

There is no way the Monopole problem can be regarded as merely a feature. In 1979 Preskill [4] showed how quite generally a significant number of magnetic monopoles would be produced during the cosmic phase transition required by Grand Unification. Being non-relativistic ( $\rho \propto a^{-3}$ ), they would rapidly come to dominate over the ordinary matter (which was then relativistic,  $\rho \propto a^{-4}$ ). Thus it appeared that a Universe that is nearly 100% magnetic monopoles today was a robust prediction from GUT's, and thus the idea of Grand Unification was easily ruled out.

## 3 The Machinery of Inflation and what it can do

Over the years numerous inflationary “scenarios” have been proposed. My goal here is to introduce the basic ideas that underlie essentially all the scenarios.

### 3.1 The potential dominated state

Consider adding an additional scalar matter field  $\phi$  (the “inflaton”) to the cosmological model. In general the stress-energy tensor of this field will depend on the space and time derivatives of the field as well as the potential  $V(\phi)$ . In the limit where the potential term dominates over the derivative terms the stress-energy tensor of  $\phi$  takes the form

$$T_{\nu}^{\mu} = \text{Diag}(V, -V, -V, -V) \quad (7)$$

which leads (by comparison with Eqn. 3) to  $\rho = -p = V$ .

Solving Eqn. 2 gives  $\rho(a) = V$ . Because we are assuming  $\dot{\phi}$  is negligible we also have  $V = \text{const.}$  Under these circumstances  $H = \text{const.}$  and the Friedmann equation gives exponential expansion:

$$a(t) \propto e^{Ht}, \quad (8)$$

very different from the power law behavior in the SBB.



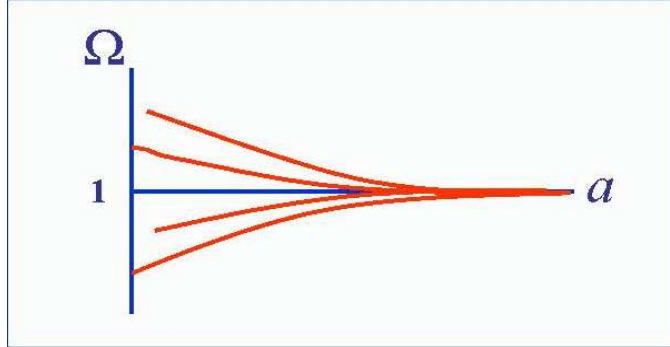


Figure 3: During inflation  $\Omega = 1$  is an attractor

### 3.2 A simple inflationary cosmology: Solving the problems

In the early days the inflationary cosmology was seen as a small modification of the SBB: One simply supposed that there was a period (typically around the GUT epoch) where the Universe temporarily entered a potential dominated state and “inflated” exponentially for a period of time. Things were arranged so that afterwards all the energy coupled out of  $V(\phi)$  and into ordinary matter (“reheating”) and the SBB continued normally.

This modification on the SBB has a profound effect on the cosmological problems. I will address each in turn:

**Flatness:** The flatness problem was phrased in terms of the tendency for the  $k$  term to dominate the  $\rho$  term in the Friedmann equation (Eqn. 1). During inflation with  $\rho = \text{const.}$  the reverse is true and the  $k$  term becomes negligible, driving the Universe *toward* critical density and  $\Omega = 1$ . This process is illustrated in Fig. 3.

**Horizon:** A period of inflation radically changes the causality structure of the Universe. During inflation, each causally connected region is expanded exponentially. A suitable amount of inflation allows the entire observed Universe to come from a region that was causally connected before inflation, as depicted in Fig. 4. Typically one needs the Universe to expand by a factor of around  $e^{60}$  to solve the Horizon and Flatness problems with GUT epoch inflation.

**Homogeneity:** Of course “solving” the Horizon problem does not guarantee a successful picture. Bringing the entire observable Universe inside one causally connected domain in the distant past can give one *hope* that the initial conditions of the SBB can be explained by physical processes. Still, one must determine exactly what the relevant physical processes manage to accomplish. While the result of inflation is clearcut in terms of the flatness, it is much less so in terms of the homogeneity. The good news is that a given inflation model can actually

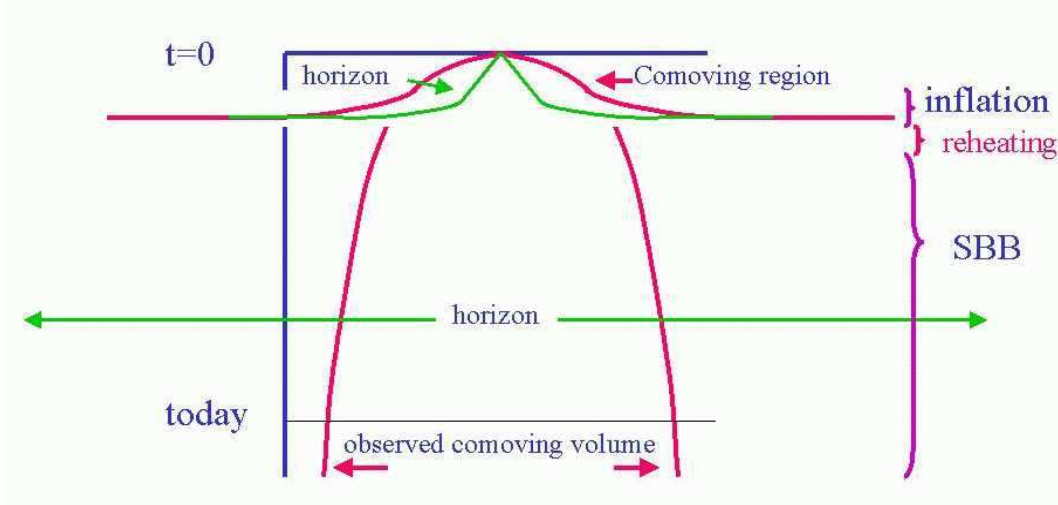


Figure 4: During inflation one horizon volume is exponentially expanded. Today, the observe region can occupy a small fraction of a causal volume.

make *predictions* for the spectrum of inhomogeneities that are present at the end of inflation. However, there is nothing intrinsic to inflation that predicts that these inhomogeneities are small. None the less, the mechanics of inflation can be further adjusted to give inhomogeneities of the right amplitude. Interestingly, inflation has a lot to say about other aspects of the early inhomogeneities, and these other aspects are testing out remarkably well in the face of new data. There will be more discussion of these matters in Sections 3.5 and 4.

**Monopoles:** The original Monopole problem occurs because GUTs produce magnetic monopoles at sufficiently high temperatures. In the SBB these monopoles “freeze out” in such high numbers as the Universe cools that they rapidly dominate over other matter (which is relativistic at that time). Inflation can get around this problem if the reheating after inflation does not reach temperatures high enough to produce the monopoles. During inflation, all the other matter (including any monopoles that may be present) is diluted to completely negligible densities and the overall density completely is dominated by  $V(\phi)$ . The ordinary matter of the SBB is created by the reheating process at the end of inflation. The key difference is that in a simple SBB the matter we see has in the past existed at all temperatures, right up to infinity at the initial singularity. With the introduction of an inflationary epoch, the matter around us has only existed up to a finite maximum temperature in the past (the reheating temperature). If this temperature is on the low side of the GUT temperature, monopoles will not be produced and the Monopole Problem is evaded.

### 3.3 Mechanisms for producing inflation

Scalar fields play a key role in the standard mechanism of spontaneous symmetry breaking, which is widely regarded in particle physics as the fundamental origin of all particle masses. Essentially all attempts to establish a deeper picture of fundamental particles have introduced additional particles to the ones observed (typically to round out representations of larger symmetry groups used to unify the fundamental forces). Conflicts with observations are then typically avoided by giving the extra particles sufficiently large masses that they could not be produced in existing accelerators. Of course, this brings scalar fields into play.

The upshot is that additional scalar fields abound, at least in the imaginations of particle theorists, and if anything the problem for cosmologists has been that there are too many different models. It is difficult to put forward any one of them as the most compelling. This situation has caused the world of cosmology to regard the “inflaton” in a phenomenological way, simply investigating the behaviors of different inflaton potentials, and leaving the question of foundations to a time when the particle physics situation becomes clearer.

The challenge then is to account for at least 60 e-folding of inflation. Looking at the GUT scale (characterized by  $M_G \approx M_P/1,000$ ) we can estimate  $H = \sqrt{(8\pi/3)M_P^{-2}M_G^4}$ , and a characteristic GUT timescale would be  $t_g \approx M_G^{-1}$ . So one could estimate a “natural” number of e-foldings for GUT-scale inflation to be  $t_g \times H \approx \sqrt{8\pi/3}(M_G/M_P)$  where I have explicitly shown the Planck Mass for clarity. The upshot is that the “natural” number of e-foldings is a small fraction of unity, and there is the question of how to get sufficient inflation.

The “sufficient inflation” issue was easily addressed in the original Guth paper [1] because the end of inflation was brought on by an exponentially long tunneling process with a timescale of the form  $t \approx M_g^{-1} \exp(B)$  where the dimensionless quantity  $B$  need take on only a modestly large value to provide sufficient inflation. In the Guth picture, the inflaton was trapped inside a classically stable local minimum of the inflation potential, and only quantum tunneling processes could end inflation. This type of potential is depicted in Fig. 5. The original picture proposed by Guth had serious problems however, because the tunneling operated via a process of bubble nucleation. This process was analyzed carefully by Guth and Weinberg [5] and it was show that reheating was problematic in these models: The bubbles formed with all the energy in their walls, and bubble collisions could not occur sufficiently rapidly to dissipate the energy in a more homogeneous way. The original model of inflation had a “graceful exit” problem that made its prediction completely incompatible with observations.

The graceful exit problem was resolved by the idea of “slow roll” inflation [6, 7]. In the slow roll picture inflation is *classically* unstable, as depicted in Fig. 6. Slow roll models are much easier to reheat and match onto the SBB. However, because the timescale of inflation is set by a classical instability, there is no exponential working to your advantage. Fine tuning of potential param-

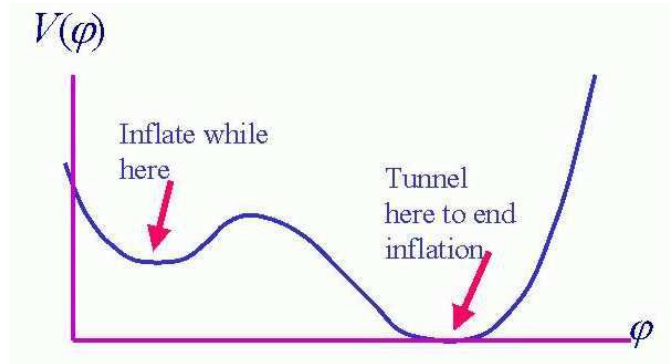


Figure 5: In the original version of inflation proposed by Guth, the potential dominated state was achieved in a local minimum of the inflaton potential. Long periods of inflation were easily produced because the timescale for inflation was set by a tunneling process. However, these models had a “graceful exit problem” because the tunneling process produced a very inhomogeneous energy distribution.

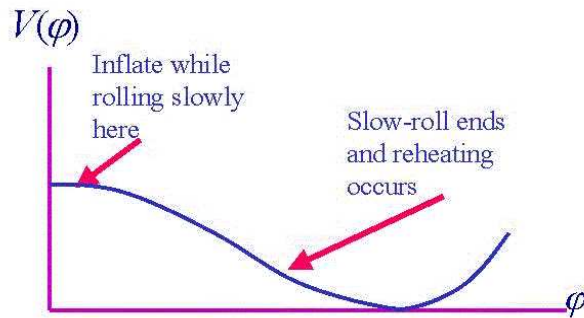


Figure 6: In “new” inflation a classical instability ends inflation. This allows for a graceful exit, but makes it more of a challenge to get sufficiently long inflation times.

eters is generally required to produce sufficient inflation in slow roll models. Essentially all current models of inflation use the slow roll mechanism. Interestingly, the constraints on the inflaton potential which are required to give the inhomogeneities a reasonable amplitude tend to exceed the constraints required to produce *sufficient* inflation. Thus the requirement of achieving sufficient inflation ends up in practice not providing any additional constraint on model building.

I have outlined the tunneling and slow roll pictures. While the slow roll remains the fundamental tool in all models, there are some variations that incorporate aspects of both. In some “open inflation” models [8] there is inflation with a tunneling instability *and* a brief period of further inflation inside the bubble after tunneling. This arrangement can produce values of  $\Omega$  which are not necessarily close to unity. Also, inflation has been studied in a multi-dimensional inflaton space where there are some barriers and some slow roll directions.

### 3.4 The start of inflation

In the original basic pictures of inflation, where a period of inflation was inserted into a SBB model, various arguments were made about why the Universe *had* to enter an inflationary phase based on symmetry restoration at high temperatures [1, 6, 7]. However, starting with a SBB *before* inflation assumes a great deal about initial conditions (which after all we are trying to explain).

Currently cosmologists that think about these things take one of two perspectives. One group tries to treat “the most random possible” initial conditions [9], and discuss how inflation can emerge from such a state of matter. The other group tries to construct the “wavefunction of the Universe” based on deep principles [10]. In some cases the wavefunction of the Universe indicates that inflation is the most likely starting point for regions that take on SBB-like behavior. I have a personal prejudice against “principles” of initial conditions, because I do not see how that gets you further than simply postulating the initial conditions of the SBB.

Coming from the random initial conditions point of view, a popular argument is that even if the probability of starting inflation is extremely small in the primordial chaos, once inflation starts it takes over the Universe. When measured in terms of volume, the exponential expansion really does appear to take over. In fact, even though there is a classical instability, the small quantum probability of remaining on the flat inflationary part of the potential is amplified by the exponential inflationary growth, and in many models most of the Universe inflates forever (so-called “eternal inflation”). So perhaps as long as inflation is possible, one does not even have to think too hard about what went before.

All of these issues are plagued by key unresolved questions (for example to do with putting a measure on the large spaces one is contemplating). I will discuss some of the open questions associated with these issues in Section 6.

### 3.5 Perturbations

The technology for calculating the production of inhomogeneities during inflation is by now very well understood (For an excellent review see [11]). The natural variables to follow are the Fourier transformed density contrasts  $\delta(k)$  in *comoving coordinates* ( $k_{co} = ak_{phys}$ ). A given mode  $\delta(k)$  behaves very differently depending on its relationship to the Hubble radius  $R_H \equiv 1/H$ . During inflation  $H = const.$  while comoving scales are growing exponentially. The modes of interest start inside  $R_H$  ( $k \gg R_H^{-1}$ ) and evolve outside  $R_H$  ( $k < R_H^{-1}$ ) during inflation. After inflation, during the SBB,  $R_H \propto t$  and  $a \propto t^{1/2}$  or  $t^{2/3}$  so  $R_H$  catches up with comoving scales and modes are said to “fall inside” the Hubble radius.

It is important to note that  $R_H$  is often called the “horizon”. This is because in the SBB  $R_H$  is roughly equal to the distance light has traveled since the big bang. Inflation changes all that, but “Hubble radius” and “horizon” are still often (confusingly) used interchangeably.

Modes that are relevant to observed structures in the Universe started out on extremely small scales before inflation. By comparison with what we see in nature today, the only excitations one expects in these modes are zero-point fluctuations in the quantum fields. Even if they start out in an excited state, one might expect that on length-scales much smaller (or energies-scales much higher) than the scale ( $\rho^{1/4}$ ) set by the energy density the modes would rapidly “equilibrate” to the ground state. These zero-point fluctuations are expanded to cosmic scales during inflation, and form the “initial conditions” that are used to calculate the perturbations today. Because the perturbations on cosmologically relevant scales must have been very small over most of the history of the Universe, linear perturbation theory in GR may be used. Thus the question of perturbations from inflation can be treated by a tractable calculation using well-defined initial conditions.

One can get a feeling for some key features by examining this result which holds for most inflationary models:

$$\delta_H^2 = \frac{32}{75} \frac{16\pi}{M_P^6} \frac{V_*^3}{(V'_*)^2}. \quad (9)$$

Here  $\delta_H^2$  is roughly the mean squared value of  $\delta$  when the mode falls below  $R_H$  during the SBB. The “\*” means evaluate the quantity when the mode in question goes outside  $R_H$  during inflation, and  $V$  is the inflaton potential. Since typically the inflaton is barely moving during inflation,  $\delta_H^2$  is nearly the same for all modes, resulting in a spectrum of perturbations that is called “nearly scale invariant”. The cosmological data require  $\delta_H^2 \approx 10^{-5}$ .

In typical models the “slow roll” condition,

$$V' M_P / V \ll 1 \quad (10)$$

is obeyed, and inflation ends when  $V'M_P/V \approx 1$ . If  $V$  is dominated by a single power of  $\phi$  during the relevant period  $V'M_P/V \approx 1$  means  $\phi \approx M_P$  at the end of inflation. Since  $\phi$  typically does not vary much during inflation  $\phi \approx M_P$  would apply throughout. With these values of  $\phi$ , and assuming simple power law forms for  $V$ , one might guess that the right side of Eqn. 9 is around unity (as opposed to  $10^{-5}$ !) Clearly some subtlety is involved in constructing a successful model.

I have noticed people sometimes make rough dimensional arguments by inserting  $V \approx M_G^4$  and  $V' \approx M_G^3$  into Eqn. 9 and conclude  $\delta_H \approx (M_G/M_P)^3$ . This argument suggests that  $\delta_H$  is naturally small for GUT inflation, and that the adjustments required to get  $\delta_H \approx 10^{-5}$  could be modest. However that argument neglects the fact that the slow roll condition must be met. When this constraint is factored in, the challenge of achieving the right amplitude is much greater than the simplest dimensional argument would indicate.

### 3.6 Further remarks

Inflation provides the first known mechanism for producing the flat, slightly perturbed initial conditions of the SBB. Despite the fact that there are many deep questions about inflation that have yet to be answered, it is still tremendous progress to know that nature has a way around the cosmological problems posed in Section 2. Whether nature has chosen this route is another question. Fortunately, inflation offers us a number of observational signatures that can help give an answer. These signatures are the subject of Sections 4 and 5.

## 4 Cosmic Coherence

In a typical inflation model the inflaton is completely out of the picture by the end of reheating and one is left with a SBB model set up with particular initial conditions. In realistic models the inhomogeneities are small at early times and can be treated in perturbation theory. Linear perturbation theory in the SBB has special properties that lead to a certain type of phase coherence in the perturbations. This coherence can be observed, especially in the microwave background anisotropies. Models where the perturbations are subject to only linear evolution at early times are called “passive” models

### 4.1 Key ingredients

At sufficiently early times the Universe is hot and dense enough that photons interact frequently and are thus tightly coupled to the Baryons. Fourier modes of the density contrast  $\delta_\gamma(k)$  in the resulting “photon-baryon fluid” have the following properties: On scales above the Jeans length the fluid experiences gravitational collapse. This is manifested by the presence of one growing and

one decaying solution for  $\delta$ . One can think of the growing solution corresponding to gravitational collapse and the decaying solution corresponding to the evolution of an expanding overdense region which sees its expansion slowed by gravity. During the radiation era Jeans length  $R_J = R_H/\sqrt{3}$  and modes “fall inside”  $R_J$  just as they fall inside  $R_H$ . For wavelengths smaller than  $R_J$  the pressure counteracts gravity and instead of collapse the perturbations undergo oscillations, in the form of pressure (or “acoustic”) waves. The experience of a given mode that starts outside  $R_J$  is to first experience gravitational collapse and then oscillatory behavior.

There is a later stage when the photons and Baryons decouple (which we can think of here in the instantaneous approximation). After decoupling the photons “free stream”, interacting only gravitationally right up to the present day. The process of first undergoing collapse, followed by oscillation, is what creates the phase coherence. I will illustrate this mechanism first with a toy model.

## 4.2 A toy model

During the “outside  $R_J$ ” regime, there is one growing and one decaying solution. The simplest system which has this qualitative behavior is the upside-down harmonic oscillator which obeys:

$$\ddot{q} = q. \tag{11}$$

The phase space trajectories are shown on the left panel in Fig. 7. The system is unstable against the runaway (or growing) solution where  $|q|$  and  $|p|$  get arbitrarily large (and  $p$  and  $q$  have the same sign). This behavior “squeezes” any initial region in phase space toward the diagonal line with unit slope. The squeezing effect is illustrated by the circle which evolves, after a period of time, into the ellipse in Fig. 7.

The simplest system showing oscillatory behavior is the normal harmonic oscillator obeying

$$\ddot{q} = -q. \tag{12}$$

This phase space trajectories for this system are circles, as shown in the right panel of Fig. 7. The angular position around the circle corresponds to the phase of the oscillation. The effect of having first squeezing and then oscillation is to have just about any phase space region evolve into something like the dotted “cigar” in the right panel. The cigar then undergoes rotation in phase space, but the entire distribution has a fixed phase of oscillation (up to a sign). The degree of phase coherence (or inverse “cigar thickness”) is extremely high in the real cosmological case because the relevant modes spend a long time in the squeezing regime.



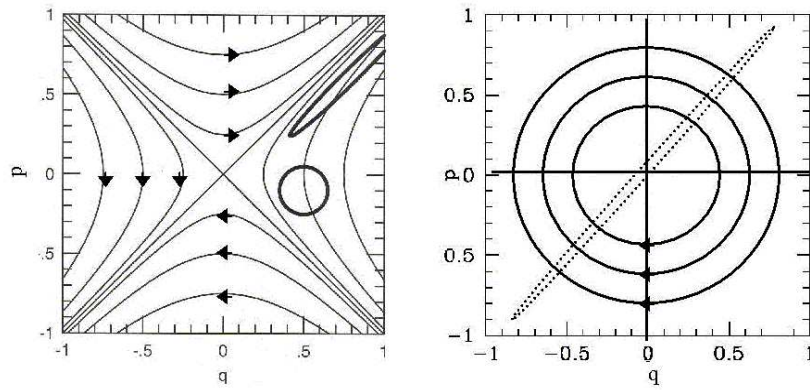


Figure 7: The phase space trajectories for an upside-down harmonic oscillator are depicted in the left panel. Any region of phase space will be squeezed along the diagonal line as the system evolves (i.e. the circle gets squeezed into the ellipse) . For a right-side-up harmonic oscillator paths in the phase space are circles, and angular position on the circle gives the phase of oscillation. Perturbations in the early universe exhibit first squeezing and then oscillatory behavior, and any initial phase space region will emerge into the oscillatory epoch in a form something like the dotted “cigar” due to the earlier squeezing. In this way the early period of squeezing fixes the phase of oscillation.

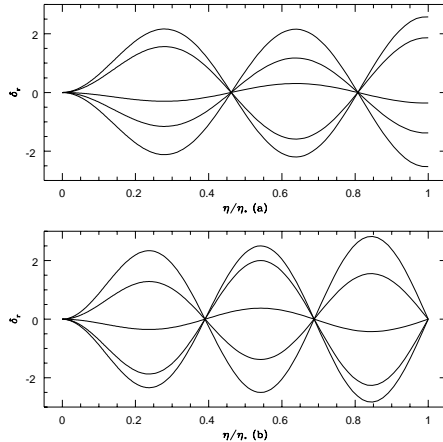


Figure 8: The evolution of  $\delta_\gamma$  for two different wavelengths (upper and lower panels) as a function of time for an ensemble of initial conditions. Each wavelength shows an initial period of growth (squeezing) followed by oscillations. The initial squeezing fixes the phase of oscillation across the entire ensemble, but different wavelengths will have different phases. The two chosen wavelengths are maximally out of phase at  $\eta_*$  the time of last scattering.

### 4.3 The results of coherence

Having presented the toy model, let us look at the behavior of the real cosmological variables. Figure 8 shows  $\delta_\gamma$  (the photon fluctuations) as a function of (conformal) time  $\eta$  measured in units of  $\eta_*$ , the time at last scattering. Two different wavelengths are shown (top and bottom panels), and each panel shows several members of an ensemble of initial conditions. Each curve shows an early period of growth (squeezing) followed by oscillation. The onset of oscillation appears at different times for the two panels, as each mode “enters”  $R_J$  at a different time. As promised, because of the initial squeezing epoch all curves match onto the oscillatory behavior at the same phase of oscillation (up to a sign). Phases can be different for different wavelengths, as can be seen by comparing the two panels.

To a zeroth approximation the event of last scattering simply releases a snapshot of the photons at that moment and sends them free-streaming across nearly empty space. The left panel of Fig. 9 (solid curve) shows the mean squared photon perturbations at the time of last scattering in a standard inflationary model, vs  $k$ . Note how some wavenumbers have been caught at the nodes of their oscillations, while others have been caught at maxima. This feature is present despite the fact that the curve represents an *ensemble average* because

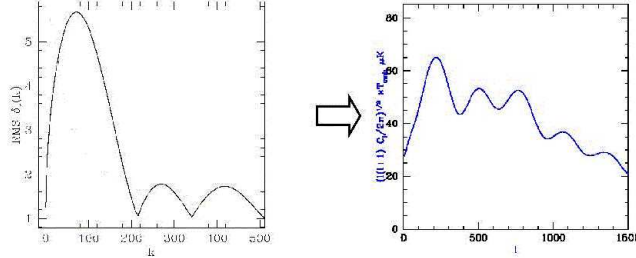


Figure 9: The left panel shows how temporal phase coherence manifests itself in the power spectrum for  $\delta$  in Fourier space, shown here at one moment in time. Even after ensemble averaging, some wavelengths are caught at zeros of their oscillations while others are caught at maximum amplitudes. These features in the radiation density contrast  $\delta$  are the root of the wiggles in the CMB angular power shown in the right panel.

the same phase is locked in for each member of the ensemble.

The right panel of Fig. 9 shows a typical angular power spectrum of CMB anisotropies produced in an inflationary scenario. While the right hand plot is not exactly the same as the left one, it is closely related. The CMB anisotropy power is plotted vs. angular scale instead of Fourier mode, so the x-axis is “ $l$ ” from spherical harmonics rather than  $k$ . The transition from  $k$  to  $l$  space, and the fact that other quantities besides  $\delta_\gamma$  affect the anisotropies both serve to wash out the oscillations to some degree (there are no zeros on the right plot, for example). Still the extent to which there *are* oscillations in the CMB power is due to the coherence effects just discussed.

As our understanding of the inflationary predictions has developed, the defect models of cosmic structure formation have served as a useful contrast [12, 13]. In cosmic defect models there is an added matter component (the defects) that behaves in a highly nonlinear way, starting typically all the way back at the GUT epoch. This effectively adds a “random driving term” to the equations that is constantly driving the other perturbations. These models are called “active” models, in contrast to the passive models where all matter evolves in a linear way at early times. Figure 10 shows how despite the clear tendency to oscillate, the phase of oscillation is randomized by the driving force. In all known defect models the randomizing effect wins completely and there are no visible oscillations in the CMB power. This comparison will be discussed further in the next section.

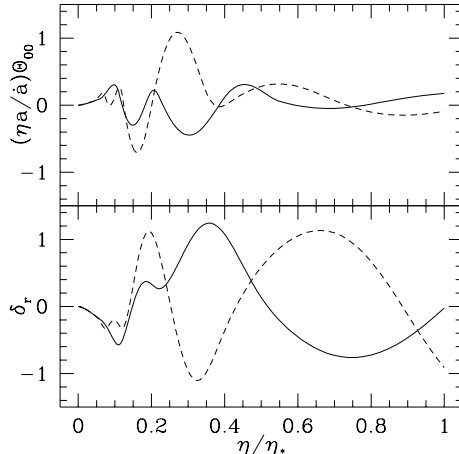


Figure 10: Decoherence in the defect models: The two curves in the upper panel show two different realizations of the effectively random driving force for a particular mode of  $\delta_\gamma$ . The corresponding evolution of  $\delta_\gamma$  is shown in the lower panel. There is no phase coherence from one member of the ensemble to the next.

## 5 Predictions

Given a region that has started inflating, and given the inflaton potential and the coupling of the inflaton to other matter, very detailed and unambiguous predictions can be made about the initial conditions of the SBB epoch that follows inflation. The catch is that at the moment we do not have a unique candidate for the inflaton and furthermore we are pretty confused in general about physics at the relevant energy scales.

Despite these uncertainties inflation makes a pretty powerful set of predictions. I will discuss these predictions here, but first it is worth discussing just what one might mean by “powerful” predictions. From time to time informal discussions of inflation degenerate into a debate about whether inflation will ever *really* be tested. The uncertainties about the inflaton mentioned above are the reason such debates can exist. As long as there is no single theory being tested, there is “wobble room” for the theory to bend and adapt to pressure from the data.

Today there are certain broad expectations of what the inflaton sector might look like, and despite their breadth, these expectations lead to a set of highly testable predictions. However, radical departures from these broad expectations are also possible, and some will argue that such departures mean that one is never really testing inflation.

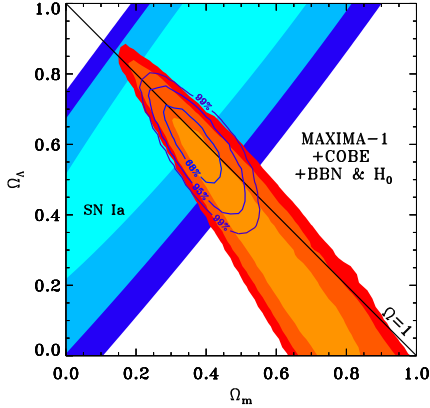


Figure 11: This is Fig. 4 from [14]. This shows that the Maxima and Supernova data consistent with a flat Universe (indicated by the diagonal  $\Omega_m + \Omega_\Lambda = 1$  line).

I have little patience for these arguments. The fundamental currency of scientific progress is the confirmation or rejections of specific ideas. There is no question that the standard picture of cosmic inflation commits itself to very specific predictions. To the extent that any one of these predictions is falsified, a real revolution will have to occur in our thinking. Furthermore, since most of these tests will be realized in the foreseeable future, the testing of inflation is not a philosophical question. It is a very real part of the field of cosmology.

To be absolutely clear on this matter, I have indicated in each topic below both the predictions of the standard picture and the possible exotic variations.

## 5.1 Flatness

**The Standard Picture:** In standard inflaton potentials a high degree of adjustment is required to achieve a suitably small amplitude for the density perturbations. It turns out that this adjustment also drives the overall time of inflation far (often exponentially) beyond the “minimal” 60 e-foldings. As a result, the total curvature of the Universe is precisely zero today in inflationary cosmologies, at least as foreseeable observations are concerned. Any evidence for a non-zero curvature at the precisions of realistic experiments will overturn the standard inflationary picture. The current data is consistent with a flat Universe as illustrated in Fig. 11.

**Exotic variations:** The possibility that an open Universe could emerge at the end of inflation has been considered for some time [15, 16] For example in [8] a period of ordinary old inflation ends by bubble nucleation, and there is

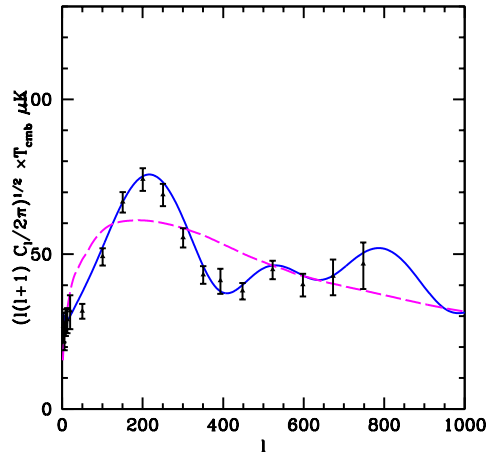


Figure 12: A current compilation of CMB anisotropy data with curves from an inflationary model (solid) and an active model (dashed). The sharp peak, a result of the phase coherence in the inflation model is impossible for realistic active models to produce.

subsequently a brief period of new inflation inside the bubble. The degree of openness is controlled by the shape of the inflaton potential, which determines the initial field values inside the bubble and the duration of inflation afterwards. The open inflation models suggest an interesting direction of study if the universe turns out to be open, but they also represent a picture that is radically different from the standard inflationary one.

## 5.2 Coherence

**The standard picture** The features in the CMB power spectrum produced by cosmic coherence (as illustrated in Fig 12) are very special signatures of inflation (and other passive models). Similar features are predicted in the polarization anisotropy power spectrum and in the polarization-temperature cross-correlations. The absence of such features would overturn this standard picture. In light of this point, the presence of the so-called “first Doppler peak” in current data (Fig. 12) [17] reflects resounding support for the standard inflationary picture. Figure 12 also shows a cosmic defect based model [18, 19] that, through some pretty exotic physics has been manipulated to attempt the creation of a first Doppler peak. Although the broad shape is reproduced, the sharp peak and corresponding valley are not.

**Exotic variations:** It has been shown that under very extreme conditions, active perturbations could mimic the features produced by coherence in the

inflationary models [20]. These “mimic models” achieved a similar effect by having the active sources produced impulses in the matter that are highly coherent and sharply peaked in time. Despite their efforts to mimic inflationary perturbations, these models have strikingly different signatures in the CMB polarization, and thus can be discriminated from inflationary models using future experiments. Also, there is no known physical mechanism that could produce the active sources required for the mimic models to work.

Another possibility is that the perturbations *are* produced by inflation but the oscillatory features in the CMB are hidden due to features introduced by a wiggly inflaton potential. It has been shown [21] that this sort of (extreme) effect would eventually turn up in CMB polarization and temperature-polarization cross-correlation measurements.

Another possibility is that inflation solves the flatness, homogeneity and horizon problems, but generates perturbations at an unobservably low amplitude. The perturbations then must be generated by some other (presumably “active”) mechanism. Currently there is no good candidate active mechanism to play this role [19], but if there was, it would certainly generate clear signatures in the CMB polarization and temperature-polarization cross-correlation measurements.

I also note here that a time varying speed of light (VSL) has been proposed as an alternative to inflation [22, 23]. So far all known realizations of this idea have produced a highly homogeneous universe which also must be perturbed by some subsequent mechanism. The building evidence against active mechanisms of structure formation are also weakening the case for the VSL idea.

### 5.3 Perturbation spectrum and other specifics

One clear prediction from the standard picture is the “nearly scale invariant” spectrum ( $\delta_H(k) \approx \text{const.}$ ) discussed in section 3.5. Strong deviations (more than several percent) from a scale invariant spectrum would destroy our standard picture. Fortunately for inflation, the current CMB and other cosmological data gives substantial support to the idea that the primordial spectrum was indeed close to scale invariant. Current data is consistent with a nearly scale invariant spectrum, as illustrated for example in Fig 13.

Also, the creation of gravitational waves during inflation is a unique effect [24]. While gravity waves from inflation are only observable with foreseeable experiments for certain inflation models [25], we know of no other source of a similar spectrum of gravity waves. Thus, if we do observe the right gravity wave spectrum this would be strong evidence for inflation.

There are a host of specific details of the perturbations that are not uniquely specified by all “Standard Picture” models. Some models have significant contributions from tensor perturbations (gravity waves) while others do not. The inflaton potential specifies the ratio of scalar to tensor amplitudes, but this ratio can be different for different models.

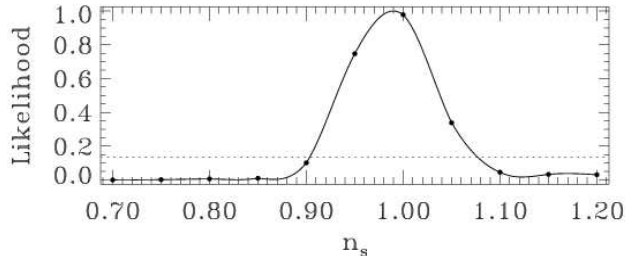


Figure 13: This is Fig. 2 from [14]. This shows that the Maxima data are consistent with a scale invariant spectrum ( $n = 1$ ). Other data show a similar level of consistency.

## 6 The Big Picture and Open Questions

There are a number of interesting open questions connected with inflation:

**The origin of the Inflaton:** It is far from clear what the inflaton actually is and where its potential comes from. This is intimately connected with the question of why the perturbations have the amplitude and spectrum they do. Currently, there is much confusion about physics at the relevant energy scales, and thus there is much speculation about different possible classes of inflaton potentials. One can hope that a clearer picture will eventually appear as some deeper theory (such as string/M theory) emerges to dictate the fundamental laws of physics at the inflation scale.

**Physics of the inflaton:** Having chosen an inflaton potential, one can calculate the perturbations produced during inflation *assuming* the relevant field modes for wavelengths much smaller than the Hubble radius are in their ground states. This seems plausible, but it would be nice to understand this issue more clearly. Also, the inflaton field often takes on values  $O(M_P)$ . Will a yet-to-be-determined theory of quantum gravity introduce large corrections to our current calculations?

**The cosmological constant problem:** A very important open question is linked with the cosmological constant problem [26, 27, 28]. The non-zero potential energy of the inflaton during inflation is very similar to a cosmological constant. Why the cosmological constant is extremely close to zero today (at least from a particle physicist's point of view) is perhaps the deepest problem in theoretical physics. One is left wondering whether a resolution of this problem could make the cosmological constant and similar contributions to Einstein's equations identically zero, thus preventing inflation from every occurring. Interestingly, current data is suggesting that there is a non-zero cosmological constant today (see for example Fig. 11 which shows  $\Omega_\Lambda = 0$  is strongly excluded). Cosmic acceleration today is a very confusing idea to a theorist, but it actually is helpful to inflation theory in a number of ways: Firstly it shows that



the laws of physics do allow a non-zero cosmological constant (or something that behaves in a similar way). Also, it is only thanks to the non-zero  $\Omega_\Lambda$  that the current data are consistent with a flat Universe (see Fig. 11).

**Wider context and measures:** We discussed in Section 3.4 various ideas about how an inflating region might emerge from a chaotic start. This is a very challenging concept to formulate in concrete terms, and not a lot of progress has been made so far. In addition, the fact that so many models are “eternally inflating” makes it challenging to define a unique measure for the tiny fraction of the universe where inflation actually ends. It has even been argued [29, 30, 31] that these measure problems lead to ambiguities in the ultimate predictions from inflation.

In fact it is quite possible that the “chaotic” picture will extend to the actual inflaton as well [32, 33]. Fundamental physics may provide *many* different flat directions that could inflate and subsequently reheat, leading to many different versions of the “Big Bang” emerging from a chaotic start. One then would have to somehow figure out how to extract concrete predictions out of this apparently less focused space of possibilities.

I am actually pretty optimistic that in the long run these measure issues can be resolved [34]. When there are measure ambiguities it is usually time to look carefully at the actual physics questions being posed... that is, what information we are gathering about the universe and how we are gathering it. It is the observations we actually make that ultimately define a measure for our predictions. Still, we have a long way to go before such optimistic comments can be put to the test. Currently, it is not even very clear just what space we are trying to impose a measure on.

I am a strong opponent of the so-called “anthropic” arguments. All of science is ultimately a process of addressing conditional probability questions. One has the set of all observations and uses a subset of these to determine one’s theory and fix its parameters. Then one can check if the theoretical predictions match the rest of the observations. No one really expects we can predict all observations, without “using up” some of them to determine the theory, but the goal of science is use up as few as possible, thus making as many predictions as possible. I believe that this goal must be the only determining factor in deciding *which* observations to sacrifice to determine the theory, and which to try and predict. Perhaps the existence of galaxies will be an effective observation to “use up” in constraining our theories, perhaps it will be the temperature of the CMB. All that matters in the end is that we predict as much as possible.

Thus far, using “conditions for life to exist” has proven an extremely vague and ineffective tool for pinning down cosmology. Some have argued that “there must be at least one galaxy” for life to exist [35, 36, 37], but no one really knows what it takes for life to exist, and certainly if I wanted to try and answer such a question I would not ask a cosmologist. Why even mention life, when one could just as well say “we know at least one galaxy exists” and see what else we can predict? In many cases (including [35, 36, 37]) the actual research

can be re-interpreted that way, and my quibble is really only with the authors' choice of wording. It is these sorts of arguments (carefully phrased in terms of concrete observations) that could ultimately help us resolve the measure issues connected with inflation.

## 7 Conclusions

In theoretical cosmology Cosmic Inflation sits at the interface between the known and unknown. The end of inflation is designed to connect up with the Standard Big Bang model (SBB) which is highly successful at describing the current observations. Before inflation lies a poorly understood chaotic world where apparently anything goes, and even the laws of physics are not well understood.

Given that inflation is straddling these two worlds, it is not surprising that many open questions exist. Despite these open questions, inflation does a strikingly good job at addressing simpler questions that once seemed deep and mysterious when all we had was the SBB to work with. In particular, inflation teaches how it is indeed possible for nature to set up the seemingly highly “unnatural” initial conditions that are required for the SBB. Furthermore, inflation gives us a set of predictive signatures which lie within scope of realistic tests.

Already the confrontation with current observations is strongly favoring inflation. Upcoming large-scale surveys of the Universe promise that the pace of these confrontations will pick up over the next several years. Thus we should know in the foreseeable future whether we should continue to embrace inflation and build upon its successes or return to the drawing boards for another try.

## Acknowledgments

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